Plane Curvilinear Motion

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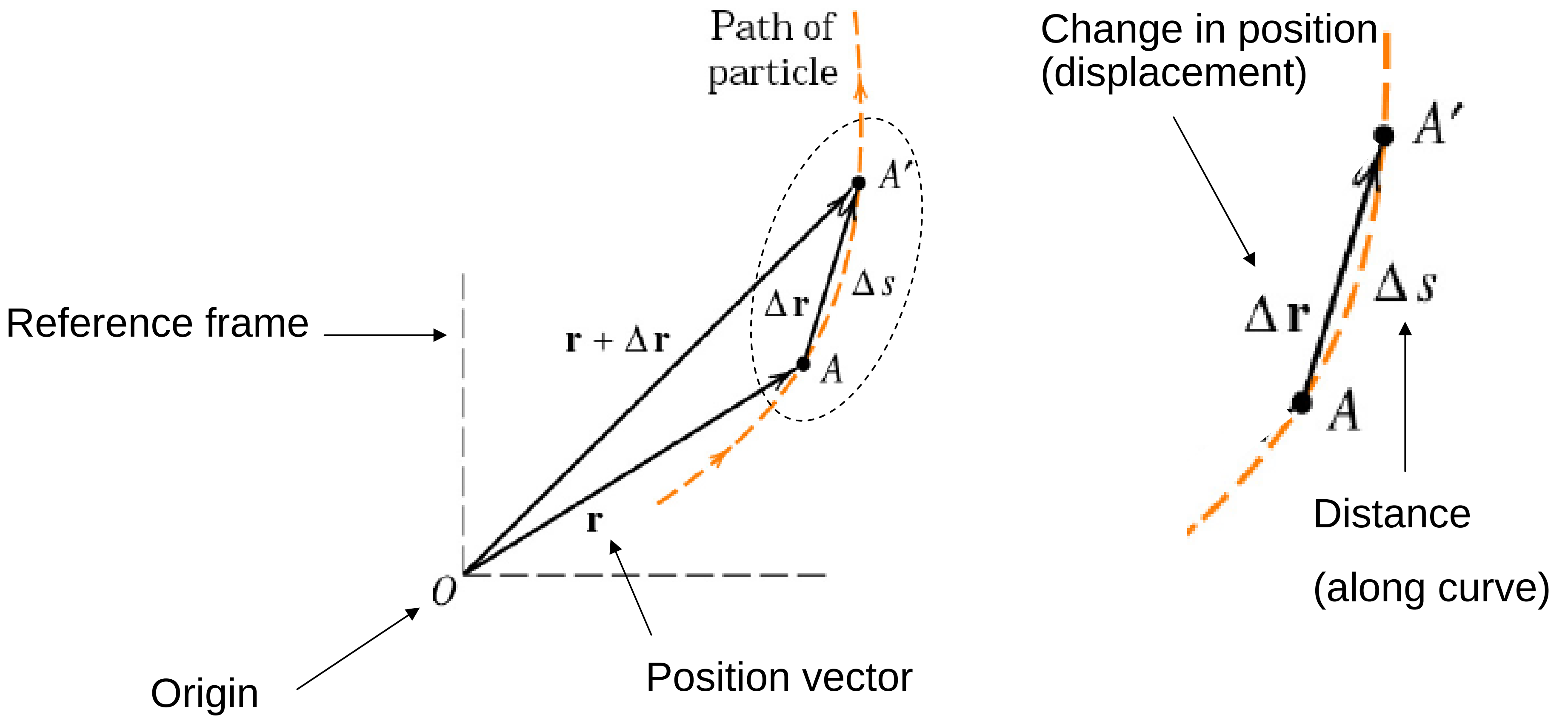
0. Introduction
1. Rectangular Coordinates (x-y)
2. Normal and Tangential Coordinates (n-t)
3. Polar Coordinates (r-θ)

Plane Curvilinear Motion

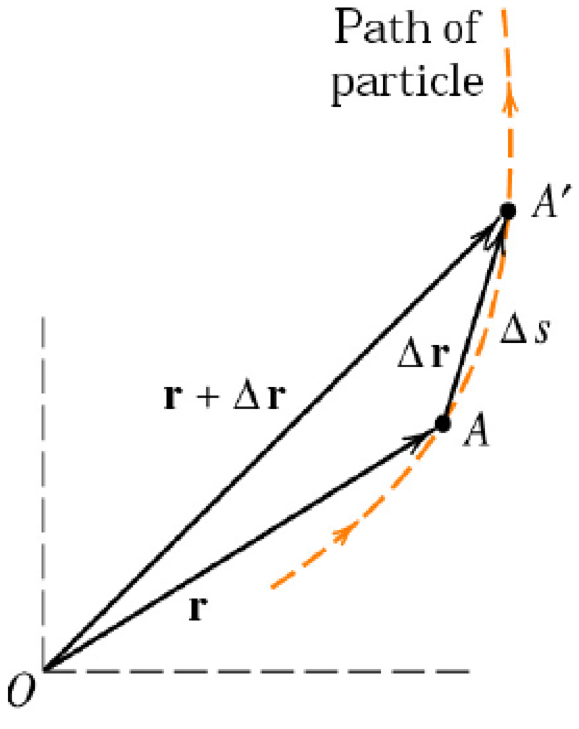
Position vector → Velocity vector → Acceleration vector

Position



Plane Curvilinear Motion

Velocity



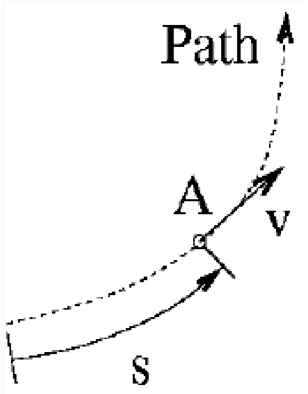
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

Magnitude: *v*

Direction: tangent to the curve at that point

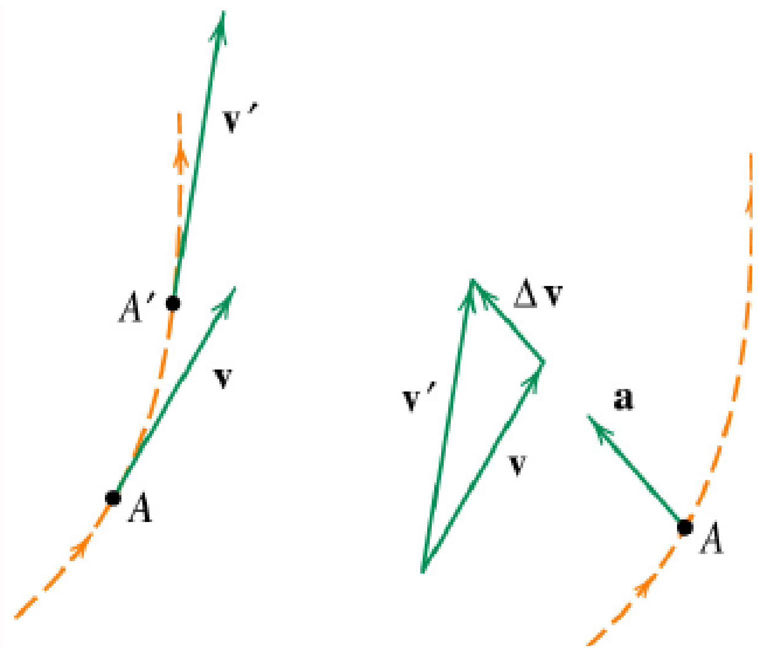
$$\vec{v} \,//\, \Delta \vec{r}$$



Note: $v \neq \dot{r}$

Plane Curvilinear Motion

Acceleration



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$$

Magnitude: *a*

Direction: pointing <u>inward</u> the curve

$$\vec{a} \,//\, \Delta \vec{v}$$

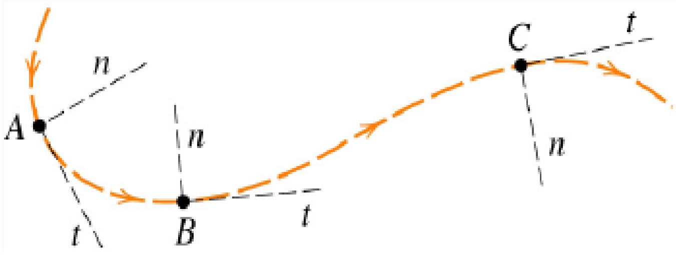
Note: $a \neq \dot{v}$

Plane Curvilinear Motion

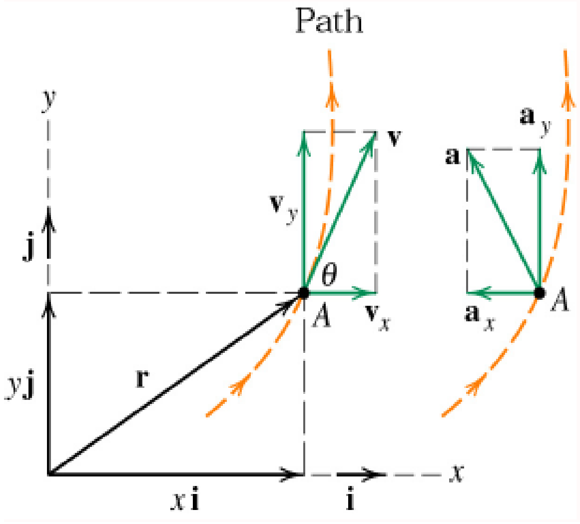
1. Rectangular Coordinates (*x*-*y*)

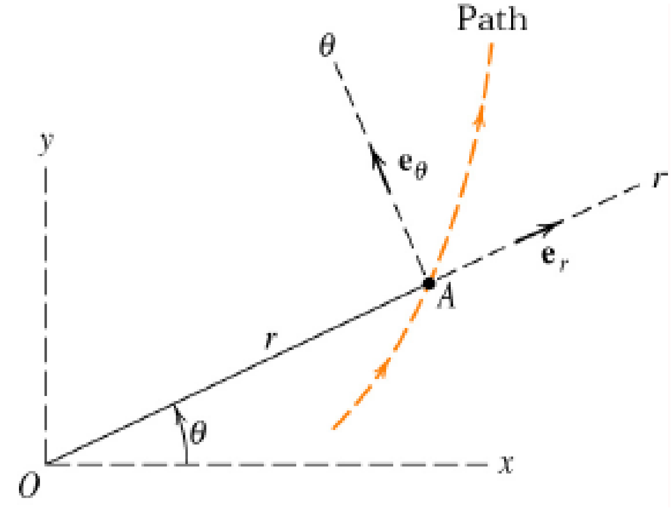
2. Normal and Tangential Coordinates (*n*-*t*)

3. Polar Coordinates (*r*-*θ*)



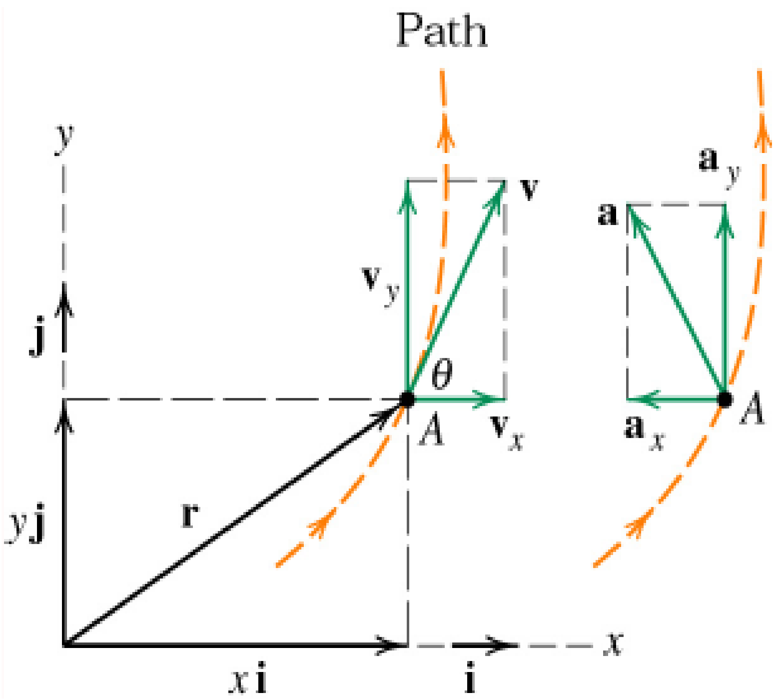
Notes: Usage will depend on the situation.
Usually, more than one system can be used.
Many times more than one system is needed at the same time





Rectangular Coordinate (*x*-*y*)

1. Rectangular Coordinates (x-y)



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = \dot{v}_x\hat{i} + \dot{v}_y\hat{j}$$

Magnitude & Direction

- Pythagoras

$$r = \sqrt{x^2 + y^2}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

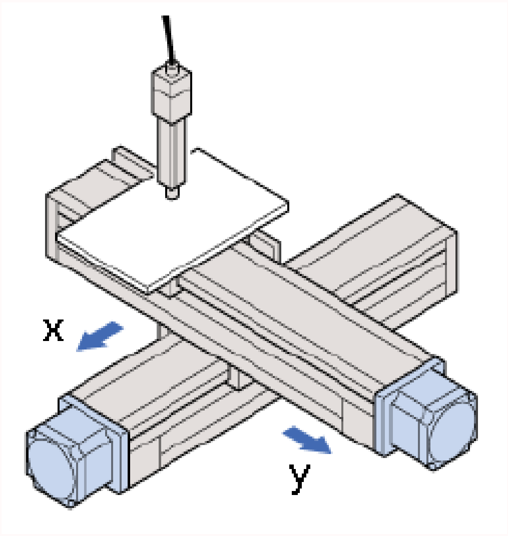
$$a = \sqrt{a_x^2 + a_y^2}$$

- Trigonometry (sine and cosine laws, etc.)

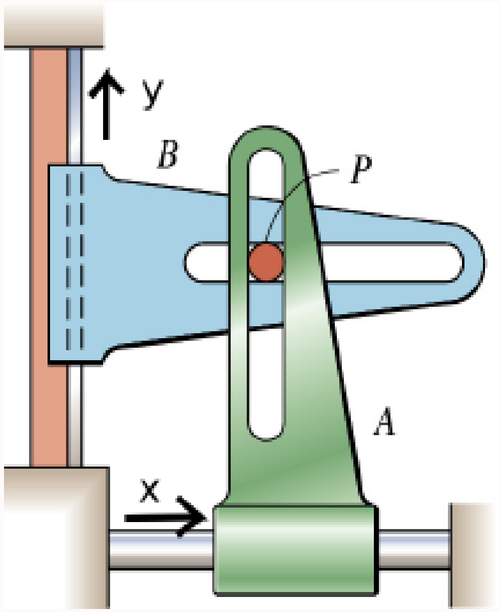
eg. $$\tan\theta = \frac{v_y}{v_x}$$

1. Rectangular Coordinates (x-y)

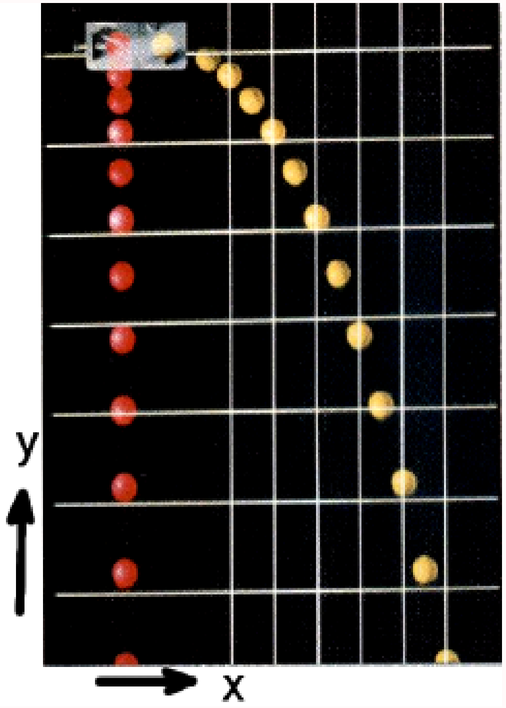
Applications



Motorized x-y table

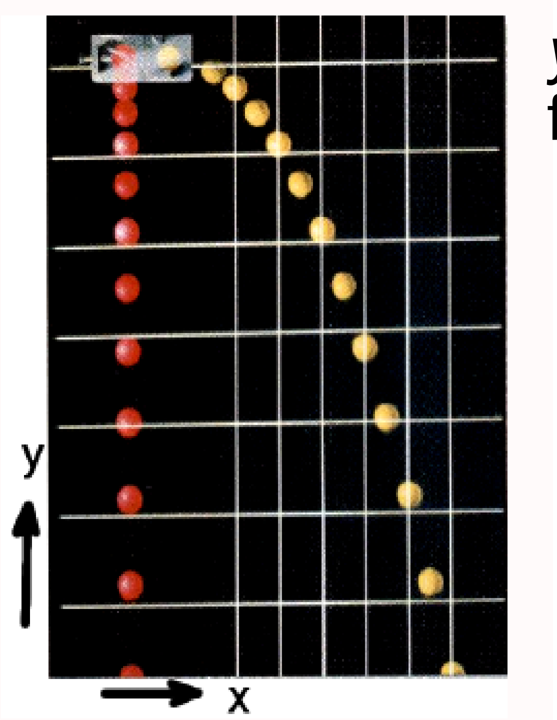


A model of the x-y table



1. Rectangular Coordinates (x-y)

Projectile Motion



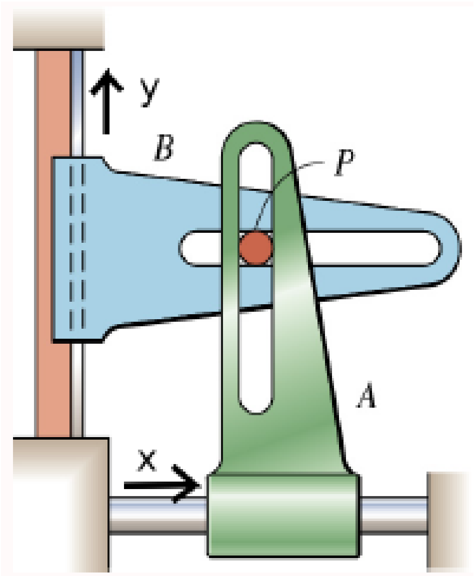
Picture of an experiment; snap shots taken at a fixed interval of time

y motion can be considered independently from the *x* direction

1. Rectangular Coordinates (x-y)

Example 1:

Suppose, at time $t = 0$, v_A = 1m/s to the right and v_B = 2m/s downward. During $t = 0$ to $t = 5$ sec, a_A = 3 m/s^2 to the left and a_B = 1 m/s^2 downward, find $\vec{v}_P$ at $t = 5$ sec.



Notes:

Notice that *x*-*y* coordinate is good when the motion in *x* and *y* direction can be calculated separately.

Techniques of rectilinear motion can be applied to motion in each of the two directions.

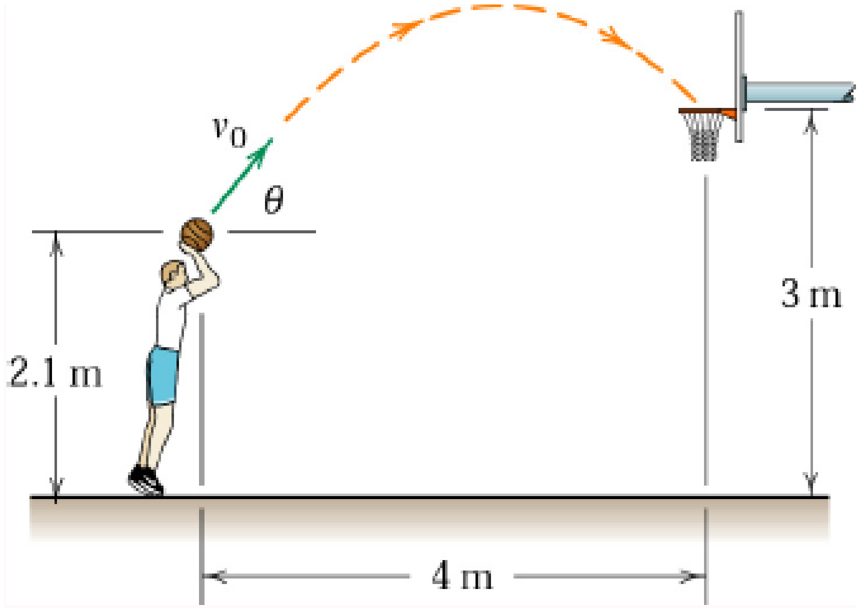
Make sure you clearly define the coordinate system used and show *x* and *y* directions on the picture clearly.

Ans: $\vec{v}_P = -14\,\hat{i} - 7\,\hat{j}$

1. Rectangular Coordinates (x-y)

Example 2: Projectile Motion

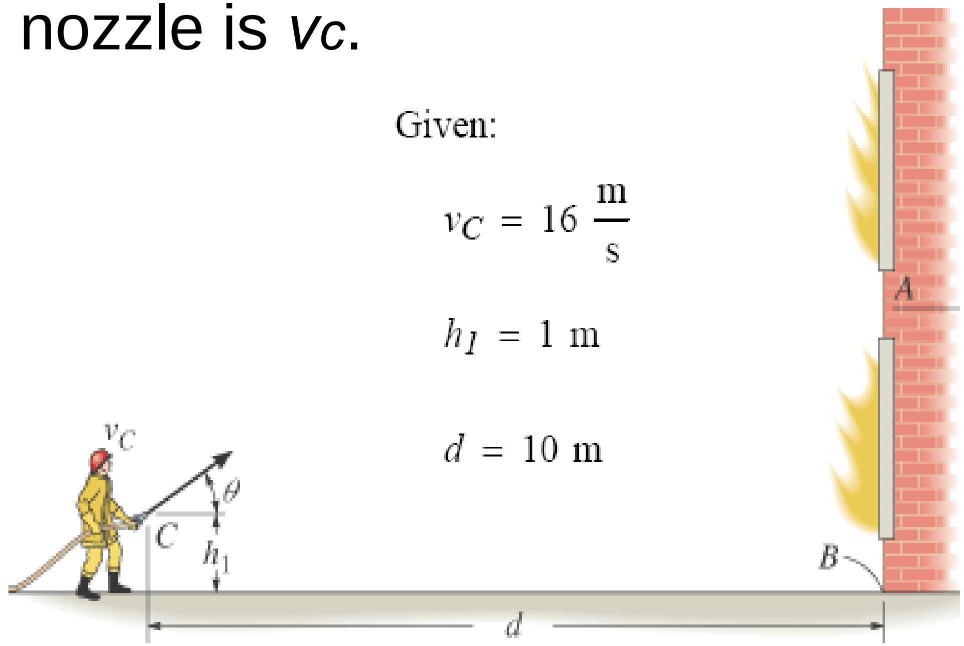
The basketball player likes to release his foul shots at an angle $\theta = 50^\circ$ to the horizontal as shown. What initial speed v_0 will cause the ball to pass through the center of the rim?



Ans: $v_0 = 7$ m/s

Example 3: Projectile Motion

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at *B*. The speed of the water at the nozzle is v_C.



Ans: $\theta = 5.33\ deg$