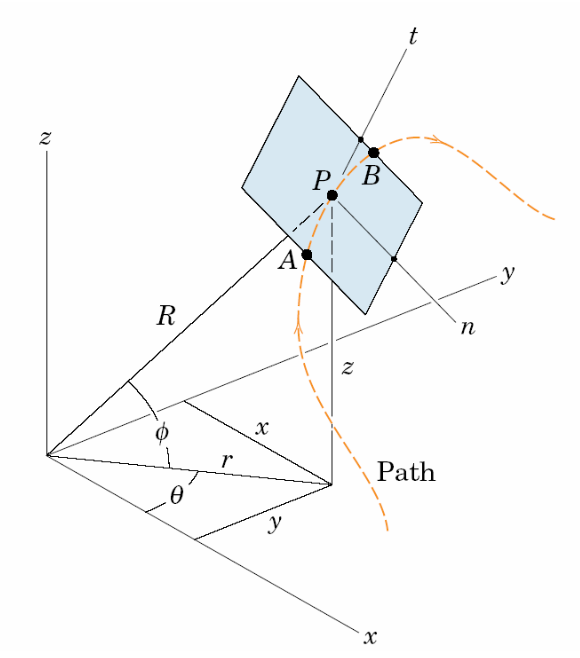
## Chapter 2
## Kinematics of Particles

# Motions and Coordinates

- Motion
  - Constrained motion
  - Unconstrained motion
- Coordinates
  - Used to describe the motion of particles

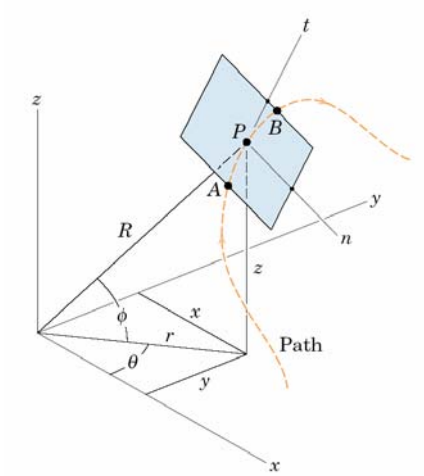


# Motion

Rectilinear motion (1-D)

Plane curvilinear motion (2-D)

Space curvilinear motion (3-D)




# Coordinates

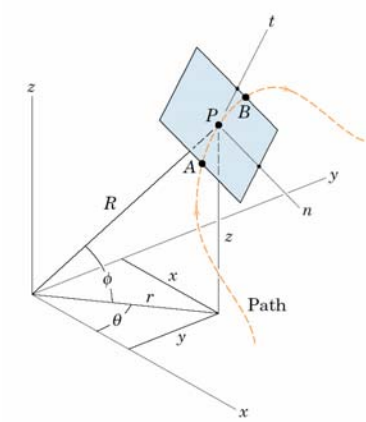
Rectangular (Cartesian) coordinates $(x,y),(x,y,z)$

Normal and tangential coordinates $(n-t)$

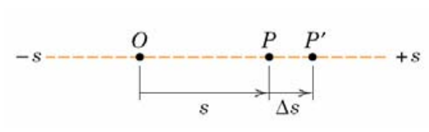
Polar coordinates $(r,\theta)$

Cylindrical coordinates $(r,\theta,z)$

Spherical coordinates $(r,\theta,\phi)$



# Chapter 2-2. Rectilinear Motion

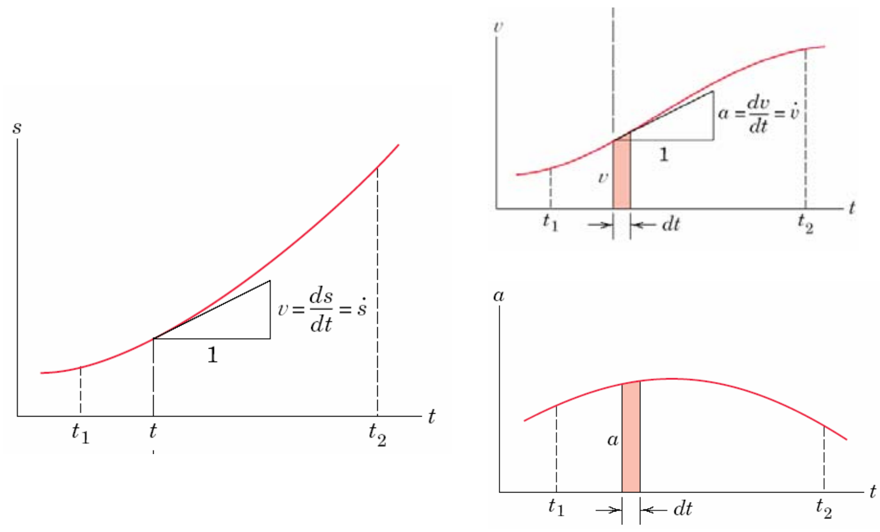


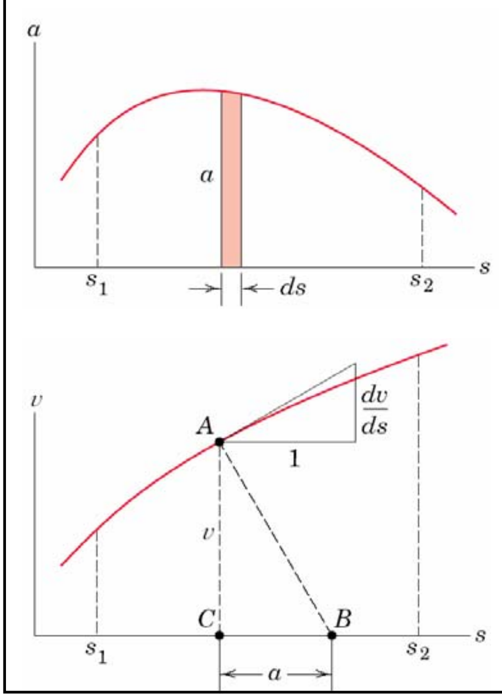

Instantaneous velocity: $v = \dfrac{ds}{dt} = \dot{s}$

Instantaneous acceleration: $a = \dfrac{dv}{dt} = \dot{v} = \dfrac{d^2v}{dt^2} = \ddot{s}$

$$\Rightarrow \begin{cases} vdv = ads \\ \dot{s}d\dot{s} = \ddot{s}ds \end{cases}$$

# Graphical Interpretations






$$ads = vdv \quad \Rightarrow \quad \frac{1}{2}(v_2^2 - v_1^2) = \text{(the area)}$$

$$\frac{a}{v} = \frac{dv}{ds}$$

# High School Physics

Given $a$=constant (and $s(t_0) = s_0,\ v(t_0) = v_0,$ when $t_0 = 0$)

(1). $a = \dfrac{dv}{dt} \quad \Rightarrow \quad v = v_0 + at$

(2). $vdv = ads \quad \Rightarrow \quad v^2 = v_0^2 + 2a(s - s_0)$

(3). $v = v_0 + at = \dfrac{ds}{dt} \quad \Rightarrow \quad s = s_0 + v_0 t + \dfrac{1}{2}at^2$

# when a ≠ constant

$$a = f(t) \Rightarrow (1).\ a = \frac{dv}{dt} \Rightarrow v = v_0 + \int_o^t a dt$$

$$(2).\ v dv = a ds \Rightarrow \frac{1}{2}(v^2 - v_0^2) = \int_{s_0}^{s} a ds$$

$$(3).\ v = \frac{ds}{dt} \Rightarrow s = s_0 + \int_0^t v dt$$

$$a = f(v) \Rightarrow (1).\ a = \frac{dv}{dt} = f(v) \Rightarrow \int_{v_0}^{v} \frac{dv}{f(v)} = \int_o^t dt = t$$

$$(2).\ v dv = a ds \Rightarrow \int_{v_0}^{v} \frac{v dv}{f(v)} = \int_{s_0}^{s} ds = s - s_0$$

$$a = f(s) \Rightarrow (2).\ v dv = a ds \Rightarrow v^2 = v_0^2 + 2\int_{s_0}^{s} f(s) ds$$

$$v = g(s) \Rightarrow (3).\ v = \frac{ds}{dt} \Rightarrow \int_{s_0}^{s} \frac{ds}{g(s)} = \int_0^t dt = t$$

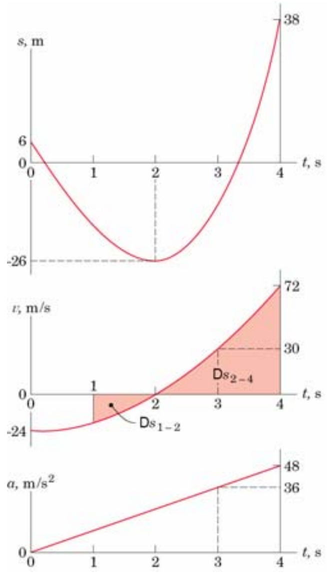
# Sample 2.1

$$s(t) = 2t^3 - 24t + 6$$

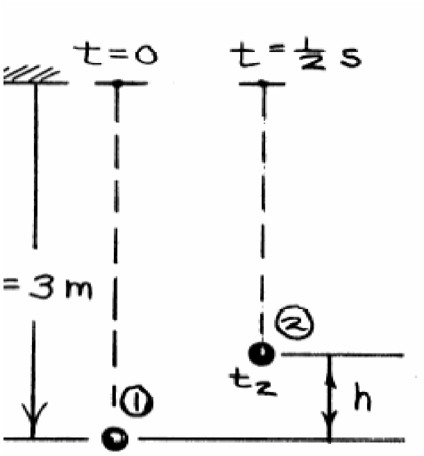
$$(1).\ v(t_1) = 72,\ t_1 = ?$$

$$(2).\ v(t_2) = 30,\ a(t_2) = ?$$

$$(3).\ s(4) - s(3) = ?$$

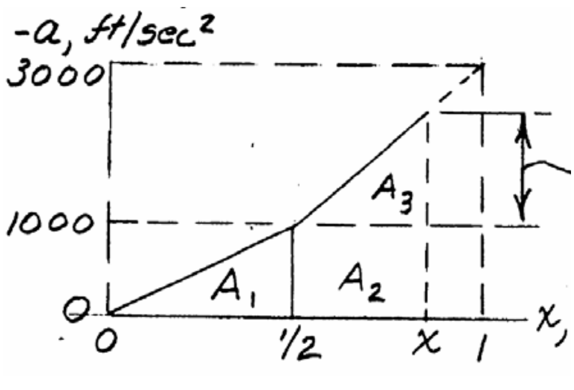


# Problem 2/19




- Small balls fall from rest through the opening at the steady rate of 2 per-second. Find the vertical displacement *h* of 2 consecutive balls when the lower one has dropped *3 m*.

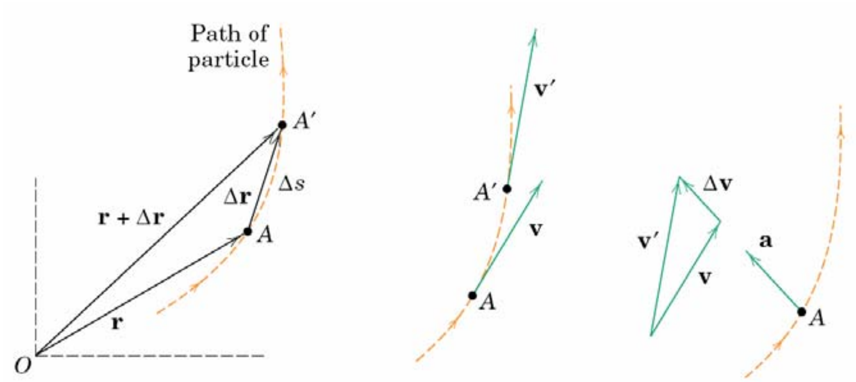
# Problem 2/50




- A bumper provides a deceleration as shown in the figure. Suppose a train is approaching the bumper at speed of 40 ft/sec.
- Determine the maximum compression of the bumper.

# Chapter 2-3. Plane Curvilinear Motion

1. 2-D motion: a special case of 3-D.
2. Define the position vector $\vec{r}$ measured from a fixed point $O$.
3. Time derivative of a position vector: $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}},\ \vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$

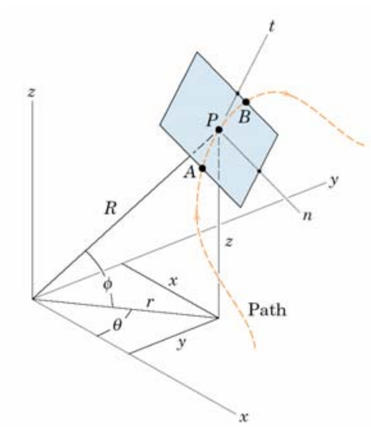



# Three coordinates systems to describe the curvilinear motion

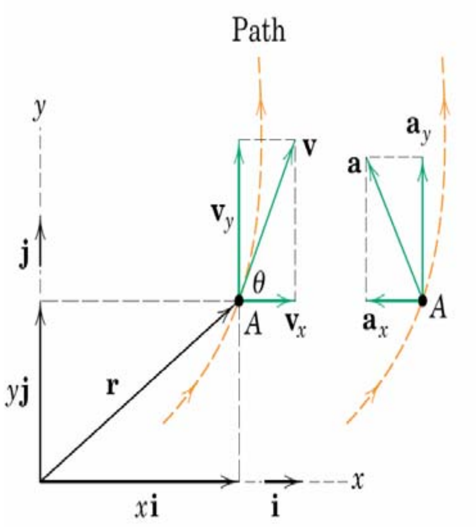
Rectangular (Cartesian) coordinates $(x, y)$

Normal and tangential coordinates $(n - t)$

Polar coordinates $(r, \theta)$



## Chapter 2.4 Rectangular coordinates (x-y)



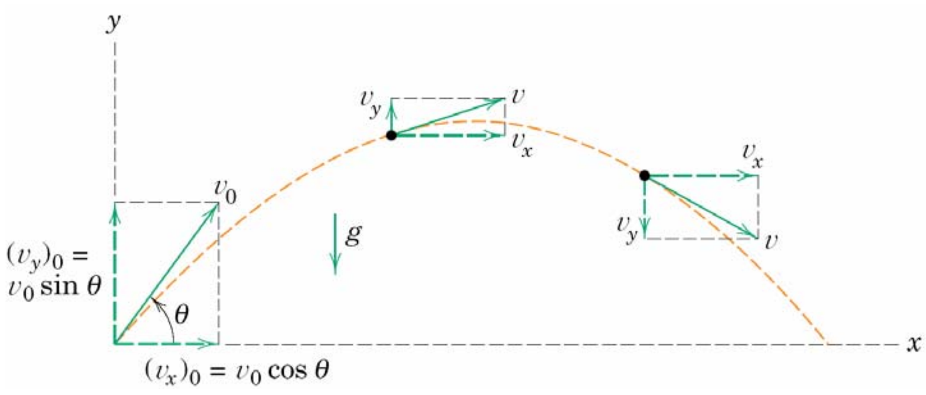

Vector representation

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$\vec{v} = \dot{\vec{r}} = \dot{x}\vec{i} + \dot{y}\vec{j}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$

## Projectile motion: $a_x = 0,\ a_y = -g$



# Sample 2.5

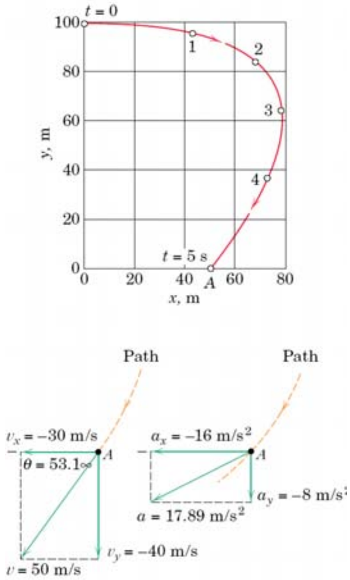
$v_x(t) = 50 - 16t,$

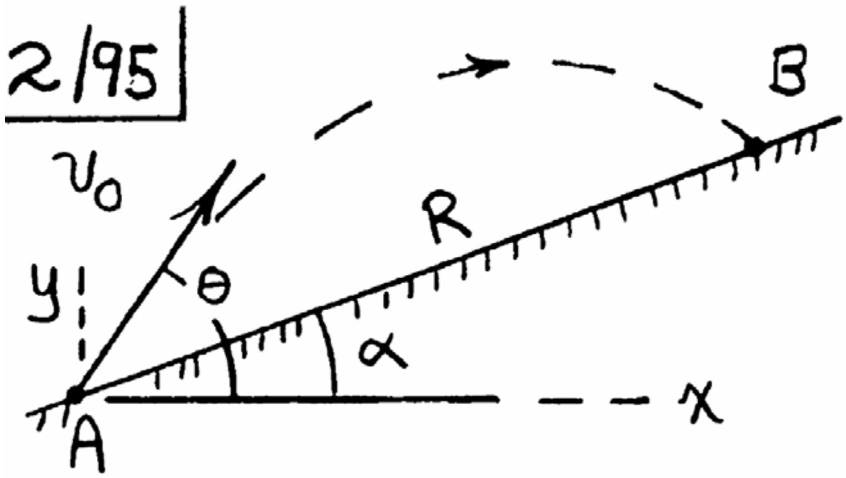
$y(t) = 100 - 4t^2.$

$x(0) = 0,\ y$ in meter and $t$ in second.

Question:

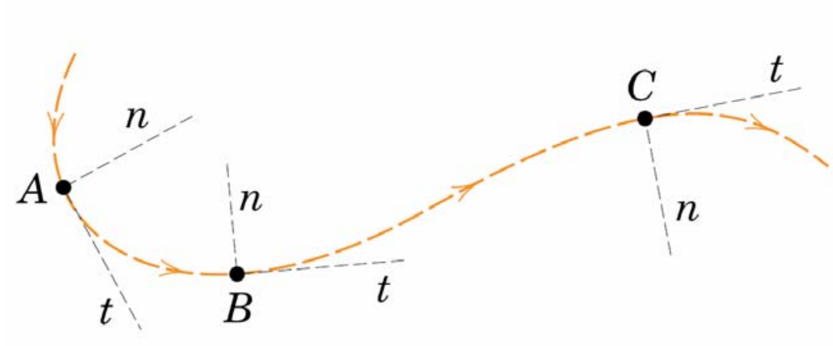
when $y(t) = 0,\ a = ?$ and $v = ?$



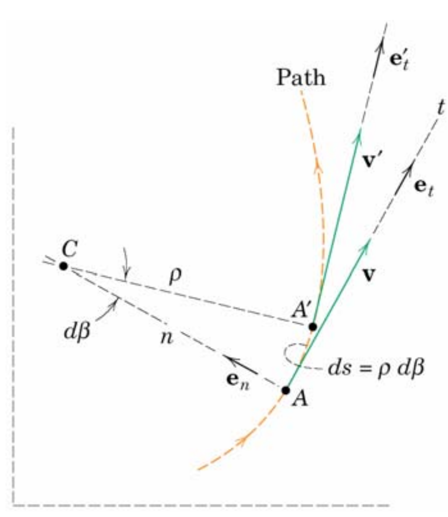



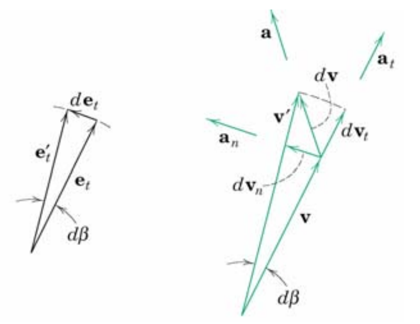

Determine $\theta$ such that $R$ is maximized.

# Chapter 2.5
# Normal and Tangential Coordinates (n-t)




- The positive direction of $n$ is always taken toward the center of curvature of the path.



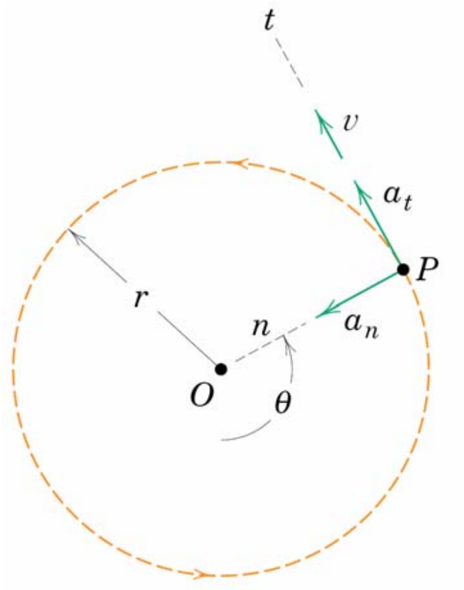

$$ds = \rho d\beta,$$

$$\Rightarrow \vec{v} = v\vec{e}_t = \frac{ds}{dt}\vec{e}_t = \rho\dot{\beta}\vec{e}_t$$

$$\Rightarrow \vec{a} = \dot{v}\vec{e}_t + v\dot{\vec{e}}_t = \dot{v}\vec{e}_t + \rho\dot{\beta}^2\vec{e}_n$$

$$= \dot{v}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$

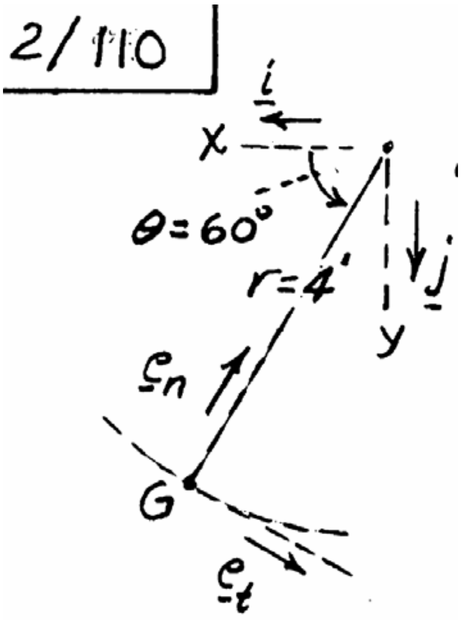
## A special case: Circular Motion




$$v = r\dot{\theta}$$

$$a_n = v\dot{\theta} = \frac{v^2}{r} = r\dot{\theta}^2$$

$$a_t = r\ddot{\theta}$$




Write the vector expression for the acceleration $a$ of the mass G of the simple pendulum in both $n$ - $t$ and $x$ - $y$ coordinates for the instance when

$\theta = 60^o$

$\dot{\theta} = 2.00$ rad/sec

$\ddot{\theta} = 4.025$ rad/sec$^2$

# Exercise 2/119

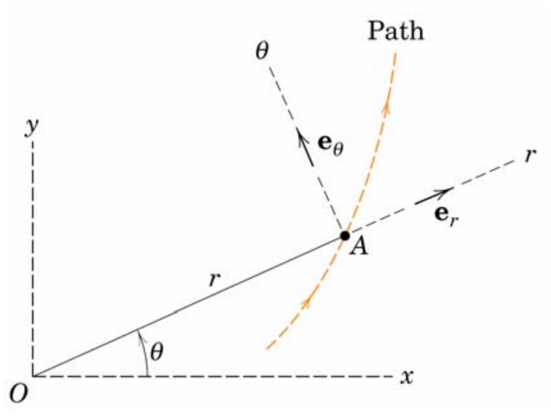
A particle moving in the $x$ - $y$ plane has the position vector as:

$$P = (\frac{3}{2}t^2, \frac{2}{3}t^3) \text{ (in)}$$

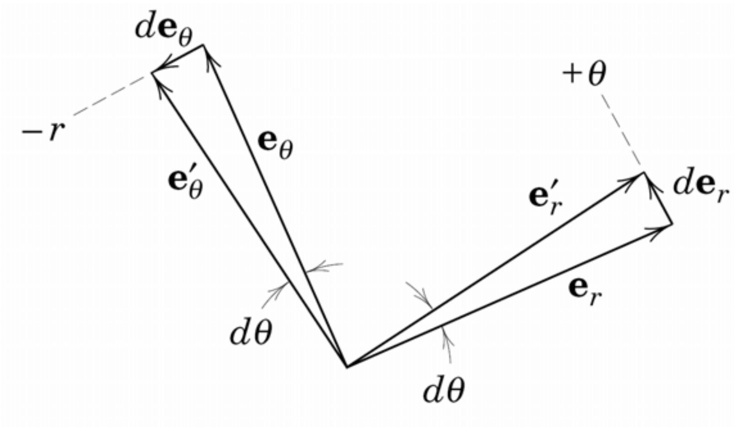
Calculate the radius of the path for the position when $t$=2 sec.

# Chapter 2.6 Polar coordinates ($r$- $\theta$ )

relative to a fixed point




$$\vec{r} = r\vec{e}_r$$

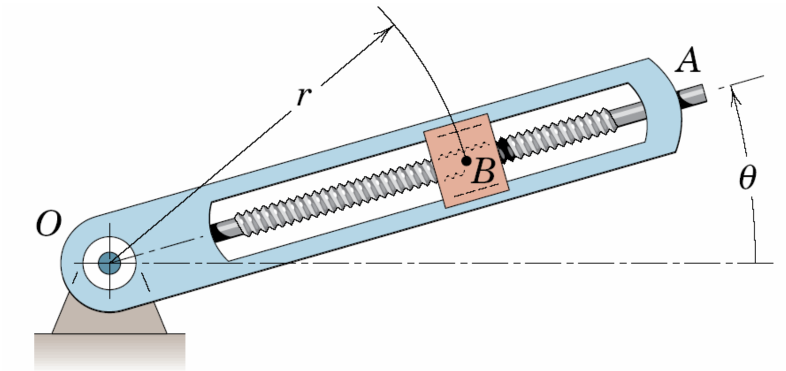



$$\vec{r} = r\vec{e}_r,$$

$$\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta,$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta.$$

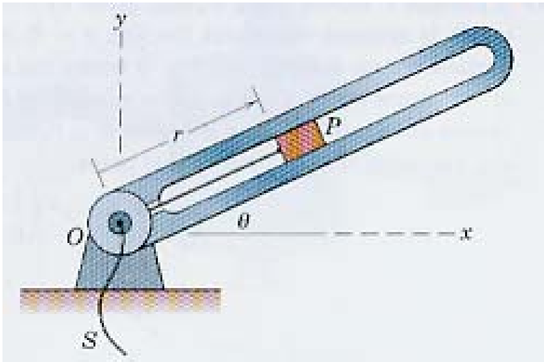
# Sample 2/9




$$\theta(t) = 0.2t + 0.02t^3, \quad r(t) = 0.2 + 0.04t^2.$$

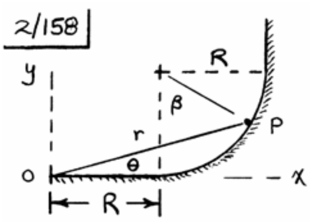
$$v(3) = ? \qquad a(3) = ?$$

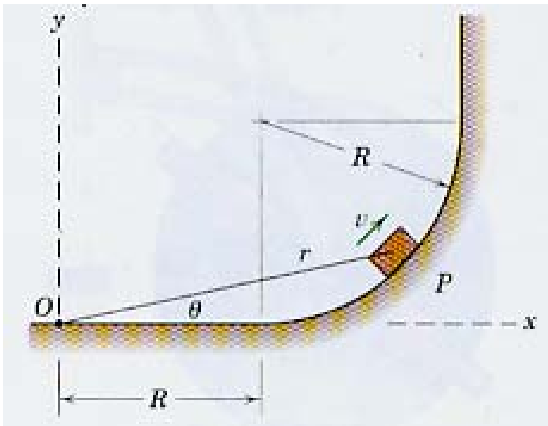
# Exercise 2/145 (slider)




$\theta(t) = 0.8t - 0.05t^2, \quad r(t) = 1.6 - 0.2t.$

$v(4) = ?$ $a(4) = ?$ and direction (relative to $x$-axis)




Constant speed

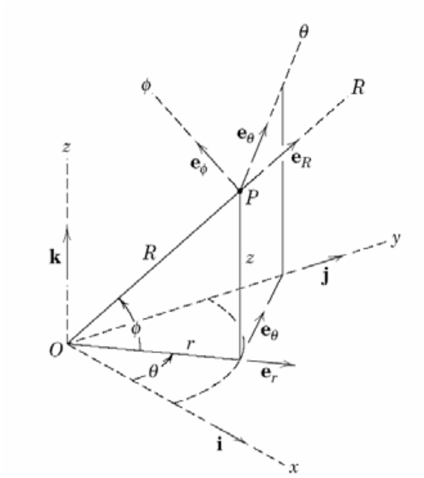
$v$=0.6 m/s

$R$=1.2 m

$r,\ \dot{r},\ \ddot{r},\ \theta,\ \dot{\theta},\ \ddot{\theta} = ?$

when $t = 2(1 + \frac{\pi}{3})$

# Chapter 2.7 Space Curvilinear Motion




- Rectangular *(x-y-z)*
- Cylindrical *(r- θ -z)*
- Spherical *(R- θ - ϕ)*

- * *n-t* coordinates

## Rectangular coordinates *(x-y-z)*

$$\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = \dot{\vec{R}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$

$$a = \dot{\vec{v}} = \ddot{\vec{R}} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$

## Cylindrical Coordinates *(r- $\theta$ -z)*

$$\vec{R} = r\vec{e}_r + z\vec{k}$$

$$\vec{v} = \dot{\vec{R}} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

$$a = \dot{\vec{v}} = \ddot{\vec{R}} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta + \ddot{z}\vec{k}$$

## Spherical Coordinates*(R- $\theta$ - $\phi$)*

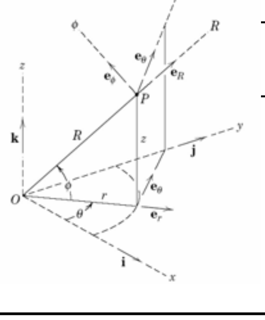
$$\vec{R} = \mathrm{R}\vec{\mathrm{e}}_R$$

$$\vec{v} = \dot{\vec{R}} = \dot{\mathrm{R}}\vec{\mathrm{e}}_r + R\dot{\theta}\cos\phi\vec{e}_\theta + R\dot{\phi}\vec{e}_\phi$$

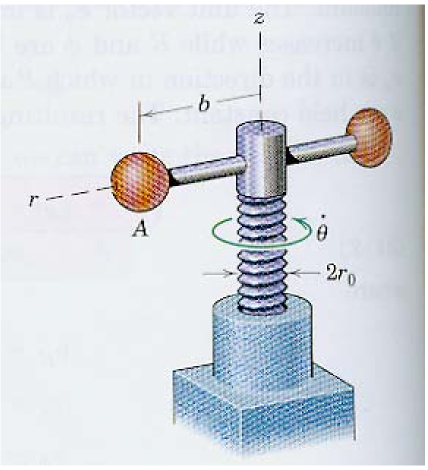
$$a = \dot{\vec{v}} = \ddot{\vec{R}} = (\ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi)\vec{e}_r$$

$$+ (R\ddot{\theta}\cos\phi + 2\dot{R}\dot{\phi}\cos\phi - 2R\dot{\phi}\dot{\theta}\sin\phi)\vec{e}_\theta$$

$$+ (R\ddot{\phi} + 2R\dot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi)\vec{e}_\phi$$



# Sample 2/11




The power screw starts from rest and is given a rotational speed $\dot{\theta}$ which increases uniformly according to $\dot{\theta} = kt$.
Suppose the lead of the screw (advancement per revolution) is L.
Determine the expression for the velocity and acceleration of the center of ball A when the screw has turned through one complete revolution from rest.

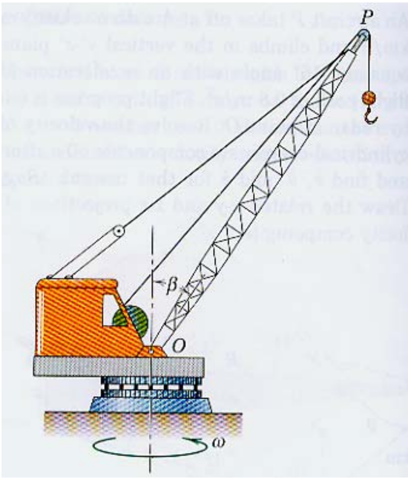
# Exercise 2/169

- The velocity and acceleration of a particle are given by

$$\vec{v} = 6\vec{x} - 3\vec{y} + 2\vec{z}$$
$$\vec{a} = 3\vec{x} - 1\vec{y} - 5\vec{z}$$

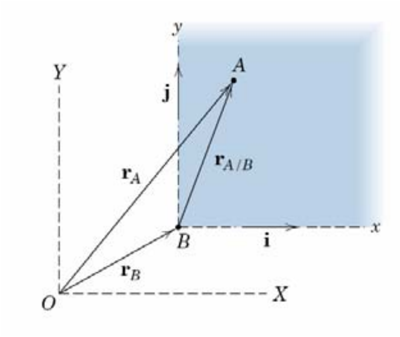
- Determine the angle between $v$ and *a*, $\dot{v}$ , and the radius of curvature.

# Exercise 2/181




The revolving crane has a boom of length 24 $m$, and is turning about the vertical axis at a constant rate of 2 $rev/\min$. At the same time, the boom is lowered at the constant rate 0.1 $rad/\sec$. Calculate the magnitudes of the velocity and acceleration of the end of the boom when it is lowered to $30^o$.

# Chapter 2.8 Relative Motion

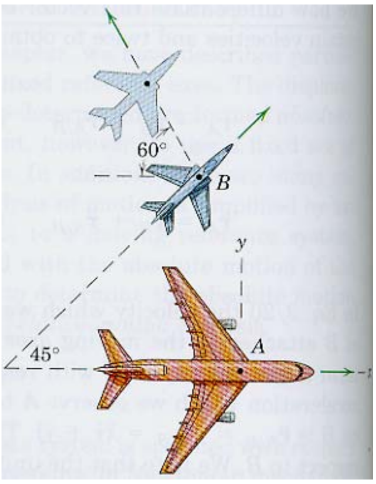



$$r_A = r_B + r_{A/B},$$

$$v_A = v_B + v_{A/B},$$

$$a_A = a_B + a_{A/B},$$

# Sample 2/12



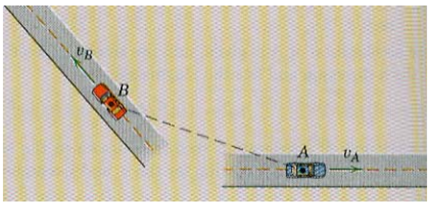

Flight A is moving east at a speed of 800 km/h

Flight B is moving northeast (45$^o$) at a speed $v$

Passengers at flight A observe that flight B moves northwest (60$^o$).

Determine $v$ = ?

# Exercise 2/188

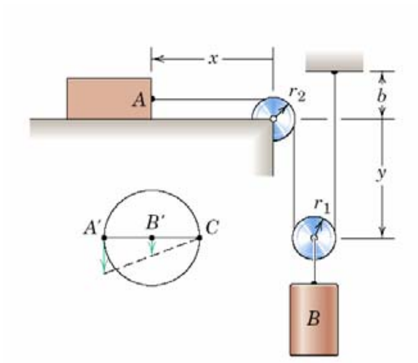



Two cars A and B are moving along straight roads. If the time rate of increase of the distance between the cars equals the magnitude of the relative velocity of the cars. What can be said concerning the velocities of the cars?

# Exercise 2/194

- A ship is capable of 16 knots through still water is to maintain a true course due west while encountering a 3-knots current running from north to south. What should be the heading of the ship (measured clockwise from the north to the nearest degree)? How long does it take the ship to proceed 24 nautical miles due west?

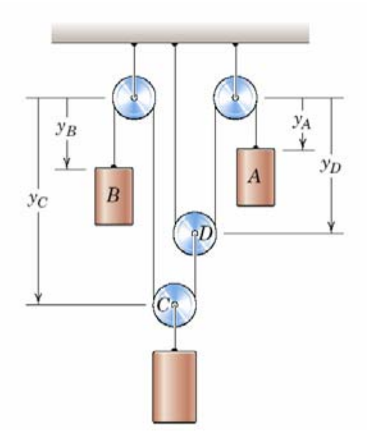
# Chapter 2.9 Constrained Motion




$$x + 2y = k$$

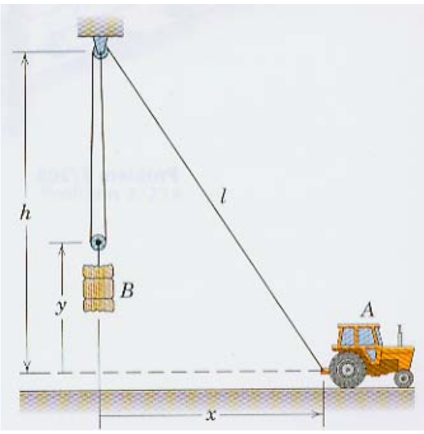
$$\dot{x} + 2\dot{y} = 0$$

$$\ddot{x} + 2\ddot{y} = 0$$



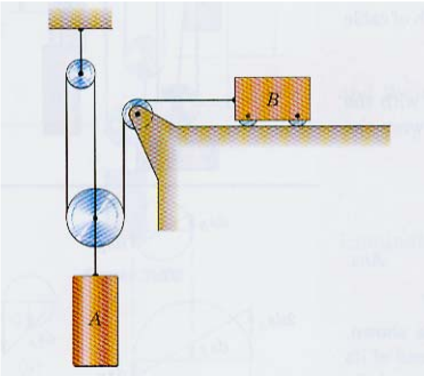

$$y_A + 2y_D = k_1$$
$$y_B - y_D + 2y_c = k_2$$
$$\dot{y}_A + 2\dot{y}_D = 0$$
$$\dot{y}_B - \dot{y}_D + 2\dot{y}_c = 0$$
$$\ddot{y}_A + 2\ddot{y}_D = 0$$
$$\ddot{y}_B - \ddot{y}_D + 2\ddot{y}_c = 0$$

# Sample 2/15



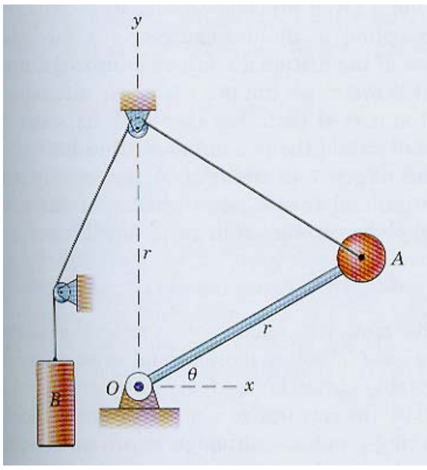

The tractor $A$ is used to hoist the bale $B$ with the pulley arrangement shown. If $A$ has a forward velocity $V_A$, determine an expression for the upward velocity $V_B$ of the bale in terms of $x$.

# Exercise 2/207




If block $B$ has a leftward velocity of 1.2 m/s, determine the velocity of cylinder $A$.

# Exercise 2/220




The particle $A$ is mounted on a loght rod pivoted at $O$ and therefore is constrained in a curvelar arc of radius $r$. Determine the velocity of $A$ in terms of the downward velocity $v_B$ of the counterweight for any angle $\theta$.

# Chapter Review

- Motion
  - Rectilinear motion (1-D)
  - Plane curvilinear motion (2-D)
  - Space curvilinear motion (3-D)

- Coordinates
  - Rectangular (Cartesian) coordinates $(x, y),(x, y, z)$
  - Normal and tangential coordinates $(n-t)$
  - Polar coordinates $(r, \theta)$
  - Cylindrical coordinates $(r, \theta, z)$
  - Spherical coordinates $(r, \theta, \phi)$