

Dynamics Review

Particle Kinematics

Basic definitions $v = \frac{dx}{dt}$ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ $v dv = a dx$

Rectilinear Motion - if acceleration constant: (these are used a lot)

$$x = \frac{1}{2} at^2 + v_0t + x_0 \quad v = at + v_0 \quad v^2 = 2a(x - x_0) + v_0^2$$

for free fall use $a = g$ downwards

for non constant acceleration: i.e. air resistance where a is proportional to v the terminal velocity is reached when $a = 0$.

Plane Curvilinear Motion

Projectile Motion: Use rectilinear motion eqns for both x and y directions with $a_x = 0$ and $a_y = -g$ with upwards being positive. The resulting motion will be a concave downwards parabola. Use Cartesian (x - y) coordinate system

Uniform Circular Motion: Acceleration is towards center $a_n = \frac{v^2}{r}$. Speed is constant. General

Circular Motion: a_n still same, but now a tangential acceleration a_t exists which can change the speed of the particle.

For all circular motion ($r = \text{constant}$) use n-t coordinate system. \mathbf{n} is always directed towards the center of curvature and \mathbf{t} is in the same direction as the velocity.

$$\mathbf{v} = v\mathbf{t} \quad \mathbf{a} = a_t\mathbf{t} + v^2/r\mathbf{n} = \dot{v}\mathbf{t} + v^2/r\mathbf{n}$$

For general motion where you have a fixed coordinate system and r is changing use polar coordinates. The \mathbf{e}_r vector always points away from the origin, and \mathbf{e}_θ is in the direction of increasing θ . The velocity is tangent to the path, but not necessarily along an axis.

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \quad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

Relative Motion

Vector relationships between two moving bodies. Use vector triangles or ijk notation.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

Constrained Motion

Two bodies connected in a way by rods, ropes or pulleys so that the motion of one effects the motion of the other.

Particle Dynamics

Newton's Second Law

Do a clear large FBD and MAD then apply Newton's second law for each mass in the problem. Use the constraints to relate the accelerations of the various bodies.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

This application will yield a series of simultaneous equations of motion which may be solved for the forces and accelerations. For rectilinear motion use Cartesian coordinates. If circular motion is involved use normal tangential coordinates with the normal axis directed from the body to the center of curvature. Use the acceleration components listed above. For general curvilinear motion polar coordinates will have to be employed. The components are also shown above.

Newton's laws are used when forces and accelerations are required in the problem.

Work & Energy

Work and energy are applied to a problem when forces, displacements and velocities are the prime variables. If no external forces are applied (or those that are applied do not do any work) conservation of energy is applied. Standard potential energies are gravitational and elastic. Energies are scalar quantities.

$V_g = mgy$ where y is the vertical distance from the datum - may be + or -

$V_e = \frac{1}{2} kx^2$ where x is the elongation of the spring from the relaxed position.

$$T = \frac{1}{2} m v^2$$

$U_{12} = \int \mathbf{F} \cdot d\mathbf{s}$ if F is constant $U = Fs \cos\theta$ where θ is the angle between \mathbf{f} and \mathbf{s} .

Impulse & Momentum

Impulse momentum is used when a problem involves forces, velocities and times.

Impulse is $\int \mathbf{F} dt$ which is $\mathbf{F}t$ if F is constant.

Momentum is $m\mathbf{v}$.

Impulse = change in momentum. This is a vector relationship so directions must be accounted for.

Collisions

Anytime a collision (impact) occurs momentum of the system will be conserved.

$$\Sigma m_i \mathbf{v}_i \text{ (before)} = \Sigma m_i' \mathbf{v}_i' \text{ (after)}$$

A totally inelastic collision is one where the objects stick together and share a common final velocity.

Energy is lost in an inelastic collision.

A perfectly elastic collision is the only collision where energy is conserved.

The coefficient of restitution (e) tells how elastic the collision was.

$$e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}} = \frac{v_2' - v_1'}{v_1 - v_2}$$

$e = 1$ for an elastic collision. $e = 0$ for a totally inelastic collision.

In a 2D collision, the coefficient of restitution only applies to the normal components of the velocities.

The tangential components remain unchanged.

Rigid Body Kinematics

Angular Motion

$$\text{Basic definitions } \omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad \omega d\omega = \alpha d\theta$$

where θ is the angular displacement, ω the angular velocity and α the angular acceleration.

$$\text{Linear counter parts } s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

For constant angular acceleration:

$$\theta = \frac{1}{2} \alpha t^2 + \omega_0 t + \theta_0 \quad \omega = \alpha t + \omega_0 \quad \omega^2 = 2\alpha(\theta - \theta_0) + \omega_0^2$$

Relative Velocity

For two points attached to the same rigid body, their velocities are related by

$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$ where the final term represents the relative velocity of A with respect to B. The vector relations will yield the proper directions.

Instantaneous Center of Zero Velocity

If the directions of two points on a rigid body are known, a point known as the instantaneous center of zero velocity for that body may be found. Draw perpendicular lines from the velocity vector. The point where they intersect is the ICZV. The angular velocity of the body is then $\omega = v_a/r_{ac}$ where r_{ac} is the distance from the point which has speed v_a to the ICZV. This works for all points in that rigid body. If two points in a rigid body have parallel velocities whose normals do not align, the body will be in translation with no angular velocity ($\omega = 0$) and all points in the body will have the same speed.

Relative Acceleration

For two points attached to the same rigid body, their accelerations are related by

$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B}$ where the second term represents the relative tangential acceleration of A with respect to B and the third term represents the relative normal acceleration of A with respect to B. The third term is valid for 2D motion. For 3D motion the full $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$ must be used.

Rigid Body Dynamics

General Comments:

For all dynamics problems involving forces, decide what type of motion is involved. Do a clear large FBD and MAD for each mass in the problem. Label all forces and acceleration vectors and label all axes.

Translation Motion

(No rotation about CG) $\alpha = 0$

Pick axes so that one lies parallel to \mathbf{a} \mathbf{a} is same for all points in body.

$$\Sigma F_{\text{parallel}} = ma \quad \Sigma F_{\text{perpendicular}} = 0 \quad \Sigma M_G = 0$$

Mass Moments of Inertia

Mass moment of inertia I is proportional to mr^2 . You should know the relations for a disk and rod.

$$\bar{I}_{\text{disk}} = \frac{1}{2} mr^2 \quad \bar{I}_{\text{rod}} = \frac{1}{12} m\ell^2 \quad I_{\text{rod about end}} = \frac{1}{3} m\ell^2$$

If radius of gyration (k) is given then $I = mk^2$. To get I about another axis use parallel axis theorem

$$I_{\text{axis}} = \bar{I} + mR^2 \quad \text{where } R \text{ is the distance from the CG to the axis.}$$

Fixed Point Rotation

Use normal - tangential coordinate system

\mathbf{n} direction towards fixed point, \mathbf{t} direction in direction of \mathbf{a}_t

Make sure α rotation is in same sense as \mathbf{a}_t

$$\mathbf{a}_t = \alpha r \quad \mathbf{a}_n = \omega^2 r \quad \omega \text{ \& } \alpha \text{ related through } \alpha d\theta = \omega d\omega$$

Take moments about fixed point and moment of inertia about fixed point.

$$\Sigma F_n = m\omega^2 r \quad \Sigma F_t = m\alpha r \quad \Sigma M_O = I_O \alpha$$

General Motion

Pick a convenient coordinate system (x - y (horizontal - vertical) often works well)

Define components of CG acceleration along axes i.e. $\mathbf{a}_G = a_x \mathbf{i} + a_y \mathbf{j}$

$$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y \quad \Sigma M_G = I_G \alpha \quad \text{Note that } a_x, a_y, \text{ and } \alpha \text{ are independent of each other.}$$

Find points of constraints and use relative acceleration equations - one for each point of constraint.

$$\mathbf{a}_A = \mathbf{a}_G + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G} \quad \mathbf{a}_B = \mathbf{a}_G + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

$$\mathbf{a}_A = a_x \mathbf{i} + a_y \mathbf{j} + \alpha \times \mathbf{r}_{A/G} - \omega^2 \mathbf{r}_{A/G} \quad \mathbf{a}_B = a_x \mathbf{i} + a_y \mathbf{j} + \alpha \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

where the directions of \mathbf{a}_A and \mathbf{a}_B are determined by the constraints on the motion. Each relative acceleration equation yields two independent equations and only brings in one more unknown.

These will give from five to seven simultaneous linear equations to solve depending on the constraints.

Work & Energy

Work - Energy is treated the same as for particles with two additions. Work can now be done by moments: $U = \int M d\theta$.

Kinetic energy now has to include rotational energy as well as translational.

$$T = \frac{1}{2} mv^2 + \frac{1}{2} \bar{I} \omega^2 \quad \text{where } \bar{I} \text{ is the centroidal moment of inertia.}$$

$$\text{For fixed point or ICZV } T = \frac{1}{2} I_P \omega^2 \quad \text{where } I_P \text{ is the moment of inertia about the fixed point or ICZV.}$$

Impulse - Momentum

Angular momentum for a rigid body is found by: $\mathbf{H} = \bar{I} \boldsymbol{\omega} + m \mathbf{r} \times \mathbf{v}$. For rotation about a fixed point this reduces to $\mathbf{H} = I_P \boldsymbol{\omega}$. Angular impulse is $\int \mathbf{M} dt$. In many problems both linear and angular impulse have to be applied.

Impact

Rotational collisions involve conservation of angular momentum about the point of impact. Usually the general angular momentum formula must be used for the initial angular momentum.