SECTION 4.1.2

•1 Three sets of functions are given. Decide whether each set is linearly dependent or independent.

$$f_1(x) = 0, \ \ f_2(x) = x, \ \ f_3(x) = x^2$$

$$f_1(x) = \sin(2x), \ \ f_2(x) = 1, \ \ f_3(x) = \sin^2 x$$

$$f_1(x) = x, \ \ f_2(x) = |x|$$

•2 For the next two sets of functions show that the given functions are independent on the given interval.

$$f_1(x) = 1 + x^3, \ \ f_2(x) = x^5, \ \ -\infty < x < \infty$$

$$f_1(x) = x, \ \ f_2(x) = \ln x, \ \ f_3(x) = x \ln x, \ \ 0 < x < \infty$$

•3 Show graphically that

$$f_1(x) = x^3 \text{ and } f_2(x) = x^2|x|$$

are linearly independent on $-\infty < x < \infty$, *and* show that the Wronskian of these two functions is identically zero on the interval.

SECTION 4.1.3

•1 Verify that $y = x^{-2}$ is a solution of $y'' = 6y^2$ on the interval $0 < x < \infty$. Show that we do *not* have a family of solutions if we write $y = cx^{-2}$ for $c \neq 0, 1$.

•2 For the next two equations, verify that the given functions form a fundamental set of solutions on the indicated interval. Form the general solution in each case.

$$y'' + 2y = 0, \;\; \sin(2x), \cos(2x), \;\; -\infty < x < \infty$$

$$x^2y'' + xy' - y = 0, \;\; \sinh(\ln x), \cosh(\ln x), \;\; 0 < x < \infty$$

•3 Verify that the solution of

$$y'' - y = \operatorname{sech} x$$

on the interval $0 < x < \infty$ is

$$y = c_1 \cosh x + c_2 \sinh x + x \sinh x - \cosh x \ln(\cosh x).$$

•4 In the previous homework set you were to show that $y = x^3$ and $y = x^2|x|$ are linearly independent functions and yet the Wronskian of these functions is identically zero. Show that both of these functions are solutions of

$$x^2y'' - 6y = 0 \text{ on } -\infty < x < \infty.$$

Explain why this does not contradict theorem 4.4.

SECTION 4.2

•1 For these next four differential equations, find a second solution of each using reduction of order. One solution is given for each equation.

$$y'' - 4y' + 4y = 0, \ \ y_1 = e^{2x}$$

$$(x^2+1)y'' - 2y = 0, \ \ y_1 = x^2 + 1$$

$$y'' + 6y' + 9y = 0, \ \ y_1 = xd^{-3x}$$

$$xy'' - (x+2)y' + 2y = 0, \ \ y_1 = e^x$$

•2 The function $y_1 = e^{-3x}$ is a solution of the homogeneous equation associated with

$$y'' + 6y' + 9y = 1.$$

Use this information to find the general solution of the equation.

SECTION 4.3

•1 Find the general solution of the following differential equations.

$$y'' - 6y = 0$$

$$y'' + 2y' - 3y = 0$$

$$y''' - 8y = 0$$

$$y''' + 5y'' + 3y' - 9y = 0$$

$$y''' + y' = 0$$

•2 Solve this differential equation subject to the given initial conditions.

$$y'' - 3y' + 2y = 0, \quad y(0) = 2, y'(0) = 3$$

•3 Find the general solution of this differential equation given that

$$y_1 = e^x \cos x \text{ is a solution.}$$

$$y''' - 4y'' + 6y' - 4y = 0$$

•4 Find a homogeneous linear differential equation with constant coefficients having the given solutions.

$$y_1 = \cos(2x), \quad y_2 = -\sin(2x)$$

SECTION 4.5

•1 Write the following differential equation in the form

$L(y) = g(x)$, where L is a differential operator with constant coefficients.

$$y''' - y'' - y' + y = e^x$$

•2 Factor this differential operator.

$$D^4 - D^3 + 4D^2 - 4D$$

•3 Verify that the given differential operator annihilates the indicated function.

$$2D^2 + \frac{1}{2}, \quad \cos\left(\frac{x}{2}\right)$$

•4 Find a differential operator which annihilates

$$x^2(1 - 2x).$$

•5 Find a differential operator which annihilates

$$x^2 + \sin x + \cos 2x.$$

•5 Find three independent functions which are annihilated by the operator

$$D^4 - 3D^3 + 4D^2 - 2D.$$

SECTION 4.6

•1 Solve the following differential equations.

$$y'' + 5y' + 4y = e^{-x} + x$$

$$y'' + 9y = \sin(3x) + 1$$

$$y'' + 9y = \sin^2 x$$

$$y''' + 2y'' + y' = (e^{2x} + e^{-2x})^2$$

•2 Solve the following equation subject to the given initial conditions.

$$y''' + 4y'' + 4y' = xe^{-2x},$$

$$y(0) = y'(0) = y''(0) = 0$$

•3 Find a particular solution for the differential equation

$$y'' + y = x\cos(2x).$$

SECTION 4.7

•1 Use variation of parameters to solve these differential equations.

$$y'' + y = -\csc x \cot x$$

$$y'' - y = \cosh x$$

$$y'' + 2y' + y = e^x$$

$$y'' - y' - 2y = 3x^2 e^{2x}$$

•2 Solve this differential equation subject to the given initial conditions.

$$9y'' - y = xe^{\frac{x}{3}}, \ \ y(0) = 1, \, y'(0) = 0$$

•3 Given that the functions

$$x^2, \ \ x^2 \ln x$$

form a fundamental set of solutions of

$$x^2y'' - 3xy' + 4y = 0 \ \text{ on } \ x > 0,$$

find the general solution of

$$x^2y'' - 3xy' + 4y = x^2.$$

•4 Given that the functions

$$\cosh(\ln x) \ \text{ and } \ \sinh(\ln x)$$

are linearly independent solutions of

$$x^2y'' + xy' - y = 0 \ \text{ for } \ x > 0,$$

find a particular solution of

$$x^2y'' + xy' - y = \operatorname{sech}(\ln x).$$

•5 Using theorem 4.9 page 148, find a particular solution of

$$y'' - 2y' + y = 4x^2 + \frac{e^x}{x}.$$

SECTION 4.5

•1 Write the following differential equation in the form

$L(y) = g(x)$, where L is a differential operator with constant coefficients.

$$y''' - y'' - y' + y = e^x$$

•2 Factor this differential operator.

$$D^4 - D^3 + 4D^2 - 4D$$

•3 Verify that the given differential operator annihilates the indicated function.

$$2D^2 + \frac{1}{2}, \quad \cos\left(\frac{x}{2}\right)$$

•4 Find a differential operator which annihilates

$$x^2(1 - 2x).$$

•5 Find a differential operator which annihilates

$$x^2 + \sin x + \cos 2x.$$

•5 Find three independent functions which are annihilated by the operator

$$D^4 - 3D^3 + 4D^2 - 2D.$$

SECTION 4.6

•1 Solve the following differential equations.

$$y'' + 5y' + 4y = e^{-x} + x$$

$$y'' + 9y = \sin(3x) + 1$$

y'' + [illegible] $= \sin^2 x$

$$y''' + 2y'' + y' = (e^{2x} + e^{-2x})^2$$

•2 Solve the following equation subject to the given initial conditions.

$$y''' + 4y'' + 4y' = xe^{-2x},$$

$$y(0) = y'(0) = y''(0) = 0$$

•3 Find a particular solution for the differential equation

$$y'' + y = x\cos(2x).$$

SECTION 4.7

•1 Use variation of parameters to solve these differential equations.

$$y'' + y = -\csc x \cot x$$

$$y'' - y = \cosh x$$

$$y'' + 2y' + y = e^{x}$$

$$y'' - y' - 2y = 3x^2 e^{2x}$$

•2 Solve this differential equation subject to the given initial conditions.

$$9y'' - y = xe^{\frac{x}{3}}, \;\; y(0) = 1, \, y'(0) = 0$$

•3 Given that the functions

$$x^2, \;\; x^2 \ln x$$

form a fundamental set of solutions of

$$x^2 y'' - 3xy' + 4y = 0 \;\text{ on }\; x > 0,$$

find the general solution of

$$x^2 y'' - 3xy' + 4y = x^2.$$

•4 Given that the functions

$$\cosh(\ln x) \;\text{ and }\; \sinh(\ln x)$$

are linearly independent solutions of

$$x^2 y'' + xy' - y = 0 \;\text{ for }\; x > 0,$$

find a particular solution of

$$x^2 y'' + xy' - y = \text{sech}\,(\ln x).$$

•5 Using theorem 4.9 page 148, find a particular solution of

$$y'' - 2y' + y = 4x^2 + \frac{e^x}{x}.$$

SECTION 6.4.1

•1 Find the singular points for these two differential equations and classify them as regular or singular.

$$y'' - \frac{1}{x+1}y' + \frac{1}{x^2}y = 0$$

$$x^2(x-1)y'' + 2xy' + (x^2-1)y = 0$$

•2 Use the method of Frobenius to find two linearly independent series solutions about the regular singular point $x = 0$ for the differential equation:

$$16(1-x)^3(1+x)x^2y'' + 32(1-x)^2(1+x)xy' + 3(1-x)^2y = 0.$$

•3 Use the method of Frobenius to solve

$$9x^2y'' + 36xy' + (9x-20)y = 0.$$

SECTION 8.1

•1 Solve the following systems of differential equations.

$$\frac{dx}{dt} - y = 0, \quad x - \frac{dy}{dt} = 1$$

$$D^2x + y = t, \quad (D-3)x + (D-3)y = 2$$

$$D^2x - 2(D^2+D)y = \cos t, \quad x + Dy = 0\ .$$

•2 Solve the following system subject to the given initial conditions.

$$\frac{dx}{dt} = -y + 1, \quad \frac{dy}{dt} = x + 2y$$

$$x(0) = 1, \quad y(0) = 0$$

SECTION 8.4.1

•1 With A, B and C given as follows, calculate the quantities AB, BA, BAC, and ABC.

$$A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$$

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

•2 With A and B as given below, find $A^T + B^T$ and $(A+B)^T$.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 9 \\ -1 & -1 \end{pmatrix}$$

•3 Write the given sum as a single column matrix.

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} t \\ -t \\ 1+t \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ t \end{pmatrix} + \begin{pmatrix} 2t \\ 1 \\ -1 \end{pmatrix}.$$

•4 With X as given below, find $\frac{dX}{dt}$.

$$X = \begin{pmatrix} \sin 2t + \cos 2t \\ 2\sin t + \cos t \end{pmatrix}$$

SECTION 8.4.2

• Use Gauss-Jordan elimination to solve the system of equations or to show that the system has no solution in each of the following three problems.

$$\begin{aligned} x + 2y + z &= 8 \\ x + y &= 3 \\ x + z &= 4 \end{aligned}$$

$$\begin{aligned} 4x + y - z &= 0 \\ x - y + 3z &= 15 \\ x + y + 7z &= 37 \end{aligned}$$

$$\begin{aligned} x + 2y + 3z &= 5 \\ 4x + 5y + 6z &= 7 \\ 7x + 8y + 9z &= 8 \end{aligned}$$

SECTION 8.4.3

• Find the eigenvalues and eigenvectors for these matrices.

$$\begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 6 & -1 & -6 \\ 3 & 0 & -2 \\ 4 & -1 & -4 \end{pmatrix}$$