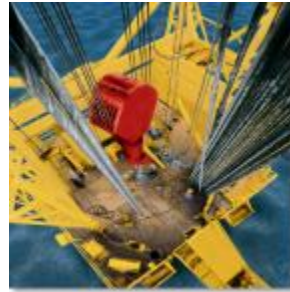


#### 4. Axial Load

### CHAPTER OBJECTIVES

- q Determine deformation of axially loaded members
- q Develop a method to find support reactions when it cannot be determined from equilibrium equations
  
- q Analyze the effects of thermal stress



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#### 4. Axial Load

### CHAPTER OUTLINE

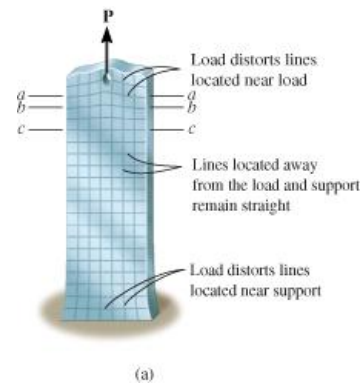
1. Saint-Venant's Principle
2. Elastic Deformation of an Axially Loaded Member
3. Principle of Superposition
4. Statically Indeterminate Axially Loaded Member
5. Thermal Stress

2

#### 4. Axial Load

##### 4.1 SAINT-VENANT'S PRINCIPLE

- q Localized deformation occurs at each end, and the deformations decrease as measurements are taken further away from the ends
- q At section  $c-c$ , stress reaches almost uniform value as compared to  $a-a$ ,  $b-b$
- $c-c$  is sufficiently far enough away from  $P$  so that localized deformation “vanishes”, i.e., minimum distance



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#### 4. Axial Load

##### 4.1 SAINT-VENANT'S PRINCIPLE

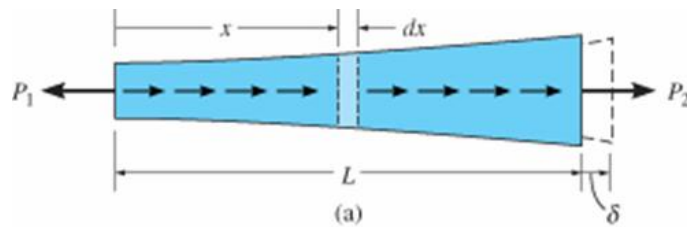
- q General rule: min. distance is at least equal to *largest dimension* of loaded x-section. For the bar, the min. distance is equal to width of bar
- q This behavior discovered by Barré de Saint-Venant in 1855, this the name of the principle
- q Saint-Venant Principle states that *localized effects* caused by any load acting on the body, will *dissipate/smooth out* within regions that are *sufficiently removed* from location of load
- q Thus, no need to study stress distributions at that points near application loads or support reactions

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#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

- q Relative displacement ( $\delta$ ) of one end of bar with respect to other end caused by this loading
- q Applying Saint-Venant's Principle, ignore localized deformations at points of concentrated loading and where x-section suddenly changes



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#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

Use method of sections, and draw free-body diagram

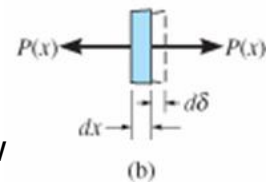
$$\sigma = \frac{P(x)}{A(x)} \quad \epsilon = \frac{d\delta}{dx}$$

- Assume proportional limit not exceeded, thus apply Hooke's Law

$$\sigma = Ee$$

$$\frac{P(x)}{A(x)} = E \left( \frac{d\delta}{dx} \right)$$

$$d\delta = \frac{P(x) dx}{A(x) E}$$

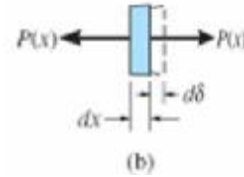


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#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\text{Eqn. 4-1} \quad \delta = \int_0^L \frac{P(x) dx}{A(x) E}$$



- $\delta$  = displacement of one pt relative to another pt
- $L$  = distance between the two points
- $P(x)$  = internal axial force at the section, located a distance  $x$  from one end
- $A(x)$  = x-sectional area of the bar, expressed as a function of  $x$
- $E$  = modulus of elasticity for material

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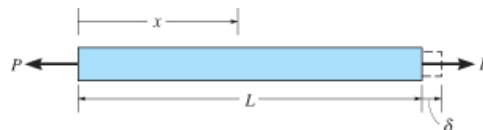
#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

###### Constant load and X-sectional area

- q For constant x-sectional area  $A$ , and homogenous material,  $E$  is constant
- q With constant external force  $P$ , applied at each end, then internal force  $P$  throughout length of bar is constant
- q Thus, integrating Eqn 4-1 will yield

$$\text{Eqn. 4-2} \quad \delta = \frac{PL}{AE}$$



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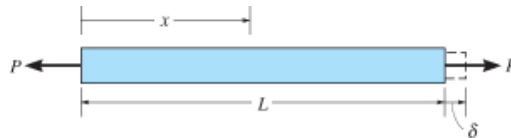
#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

###### Constant load and X-sectional area

q If bar subjected to several different axial forces, or x-sectional area or  $E$  is not constant, then the equation can be applied to each segment of the bar and added algebraically to get

$$\delta = \sum \frac{PL}{AE}$$



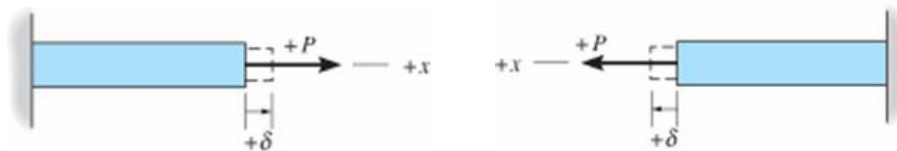
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#### 4. Axial Load

##### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

###### Sign convention

Sign	Forces	Displacement
Positive (+)	Tension	Elongation
Negative (-)	Compression	Contraction



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## 4. Axial Load

### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

#### Procedure for analysis

##### Internal force

- q Use method of sections to determine internal axial force  $P$  in the member
- q If the force varies along member's strength, section made at the arbitrary location  $x$  from one end of member and force represented as a function of  $x$ , i.e.,  $P(x)$
- q If several *constant external forces* act on member, internal force in each *segment*, between two external forces, must then be determined

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## 4. Axial Load

### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

#### Procedure for analysis

##### Internal force

- q For any segment, internal tensile force is positive and internal compressive force is negative. Results of loading can be shown graphically by constructing the normal-force diagram

##### Displacement

- q When member's x-sectional area *varies* along its axis, the area should be expressed as a function of its position  $x$ , i.e.,  $A(x)$

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## 4. Axial Load

### 4.2 ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

#### Procedure for analysis

##### Displacement

- q If x-sectional area, modulus of elasticity, or internal loading suddenly changes, then Eqn 4-2 should be applied to each segment for which the qty are constant
- q When substituting data into equations, account for proper sign for P, tensile loadings +ve, compressive -ve. Use consistent set of units. If result is +ve, elongation occurs, -ve means it's a contraction

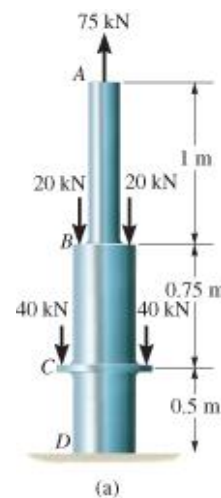
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## 4. Axial Load

### EXAMPLE 4.1

Composite A-36 steel bar shown made from two segments AB and BD. Area  $A_{AB} = 600 \text{ mm}^2$  and  $A_{BD} = 1200 \text{ mm}^2$ .

Determine the vertical displacement of end A and displacement of B relative to C.



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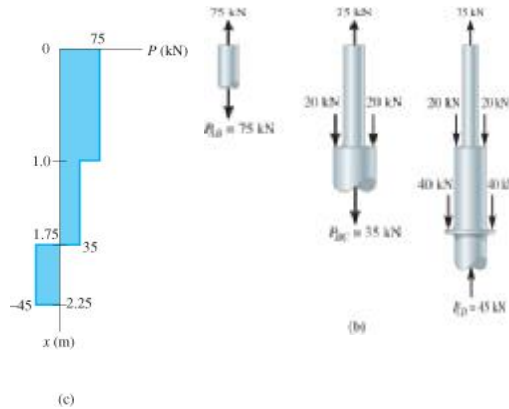
#### 4. Axial Load

#### EXAMPLE 4.1 (SOLN)

##### Internal force

Due to external loadings, internal axial forces in regions *AB*, *BC* and *CD* are different.

Apply method of sections and equation of vertical force equilibrium as shown. Variation is also plotted.



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#### 4. Axial Load

#### EXAMPLE 4.1 (SOLN)

##### Displacement

From tables,  $E_{st} = 210(10^3)$  MPa.

Use sign convention, vertical displacement of *A* relative to fixed support *D* is

$$\begin{aligned} \delta_A &= \sum \frac{PL}{AE} = \frac{[+75 \text{ kN}](1 \text{ m})(10^6)}{[600 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} \\ &\quad + \frac{[+35 \text{ kN}](0.75 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} \\ &\quad + \frac{[-45 \text{ kN}](0.5 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} \\ &= +0.61 \text{ mm} \end{aligned}$$

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#### 4. Axial Load

#### EXAMPLE 4.1 (SOLN)

##### Displacement

Since result is positive, the bar elongates and so displacement at  $A$  is upward

Apply Equation 4-2 between  $B$  and  $C$ ,

$$\begin{aligned}\delta_A &= \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{[+35 \text{ kN}](0.75 \text{ m})(10^6)}{[1200 \text{ mm}^2 (210)(10^3) \text{ kN/m}^2]} \\ &= +0.104 \text{ mm}\end{aligned}$$

Here,  $B$  moves away from  $C$ , since segment elongates

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#### 4. Axial Load

#### 4.3 PRINCIPLE OF SUPERPOSITION

- q After subdividing the load into components, the *principle of superposition* states that the resultant stress or displacement at the point can be determined by first finding the stress or displacement caused by each component load acting *separately* on the member.
- q Resultant stress/displacement determined algebraically by adding the contributions of each component

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#### 4. Axial Load

### 4.3 PRINCIPLE OF SUPERPOSITION

#### Conditions

1. The loading must be linearly related to the stress or displacement that is to be determined.
2. The loading must not significantly change the original geometry or configuration of the member

#### When to ignore deformations?

- q Most loaded members will produce deformations so small that change in position and direction of loading will be insignificant and can be *neglected*
- q Exception to this rule is a column carrying axial load, discussed in Chapter 13

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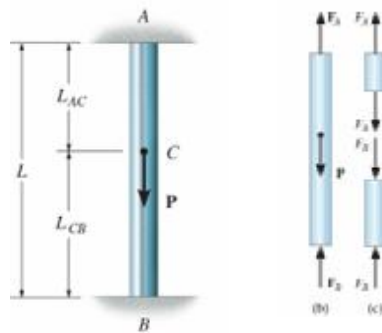
#### 4. Axial Load

### 4.4 STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

- q For a bar fixed-supported at one end, equilibrium equations is sufficient to find the reaction at the support. Such a problem is *statically determinate*
- q If bar is fixed at *both ends*, then two unknown axial reactions occur, and the bar is *statically indeterminate*

$$+\uparrow \sum F = 0;$$

$$F_B + F_A - P = 0$$



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#### 4. Axial Load

##### 4.4 STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

- q To establish addition equation, consider geometry of deformation. Such an equation is referred to as a compatibility or kinematic condition
- q Since relative displacement of one end of bar to the other end is equal to zero, since end supports fixed,

$$\delta_{A/B} = 0$$

- This equation can be expressed in terms of applied loads using a *load-displacement relationship*, which depends on the material behavior

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#### 4. Axial Load

##### 4.4 STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

- q For linear elastic behavior, compatibility equation can be written as

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

- Assume AE is constant, solve equations simultaneously,

$$F_A = P\left(\frac{L_{CB}}{L}\right) \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

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## 4. Axial Load

### 4.4 STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

#### Procedure for analysis

##### Equilibrium

- q Draw a free-body diagram of member to identify all forces acting on it
- q If unknown reactions on free-body diagram greater than no. of equations, then problem is statically indeterminate
- q Write the equations of equilibrium for the member

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## 4. Axial Load

### 4.4 STATICALLY INDETERMINATE AXIALLY LOADED MEMBER

#### Procedure for analysis

##### Compatibility

- q Draw a diagram to investigate elongation or contraction of loaded member
- q Express compatibility conditions in terms of displacements caused by forces
- q Use load-displacement relations ( $\delta = PL/AE$ ) to relate unknown displacements to reactions
- q Solve the equations. If result is negative, this means the force acts in opposite direction of that indicated on free-body diagram

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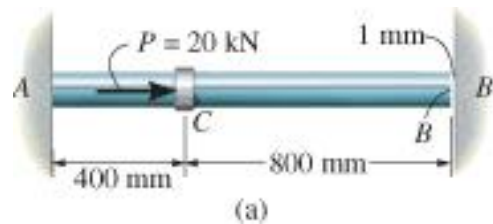
#### 4. Axial Load

#### EXAMPLE 4.5

Steel rod shown has diameter of 5 mm. Attached to fixed wall at  $A$ , and before it is loaded, there is a gap between the wall at  $B'$  and the rod of 1 mm.

Determine reactions at  $A$  and  $B'$  if rod is subjected to axial force of  $P = 20$  kN.

Neglect size of collar at  $C$ . Take  $E_{st} = 200$  GPa



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#### 4. Axial Load

#### EXAMPLE 4.5 (SOLN)

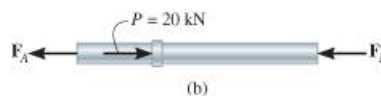
##### Equilibrium

Assume force  $P$  large enough to cause rod's end  $B$  to contact wall at  $B'$ . Equilibrium requires

$$\begin{aligned} \rightarrow +F = 0; \quad -F_A - F_B + 20(10^3) \text{ N} = 0 \end{aligned}$$

##### Compatibility

Compatibility equation:  $\delta_{B/A} = 0.001$  m



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#### 4. Axial Load

#### EXAMPLE 4.5 (SOLN)

##### Compatibility

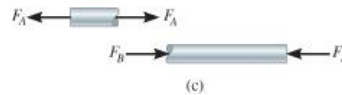
Use load-displacement equations (Eqn 4-2), apply to  $AC$  and  $CB$

$$\delta_{B/A} = 0.001 \text{ m} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE}$$

$$F_A (0.4 \text{ m}) - F_B (0.8 \text{ m}) = 3927.0 \text{ N}\cdot\text{m}$$

Solving simultaneously,

$$F_A = 16.6 \text{ kN} \quad F_B = 3.39 \text{ kN}$$



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#### 4. Axial Load

#### 4.6 THERMAL STRESS

Expansion or contraction of material is linearly related to temperature increase or decrease that occurs (for homogenous and isotropic material)

From experiment, deformation of a member having length  $L$  is

$$\delta_T = \alpha \Delta T L$$

$\alpha$  = linear coefficient of thermal expansion. Unit measure strain per degree of temperature:  $1/^\circ\text{C}$  (Celsius) or  $1/^\circ\text{K}$  (Kelvin)

$\Delta T$  = algebraic change in temperature of member

$\delta T$  = algebraic change in length of member

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#### 4. Axial Load

### 4.6 THERMAL STRESS

q For a *statically indeterminate* member, the thermal displacements can be constrained by the supports, producing thermal stresses that must be considered in design.



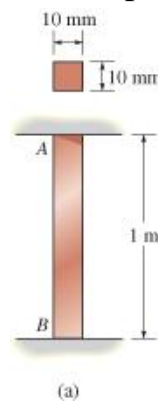
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#### 4. Axial Load

### EXAMPLE 4.10

A-36 steel bar shown is constrained to just fit between two fixed supports when  $T_1 = 30^\circ\text{C}$ .

If temperature is raised to  $T_2 = 60^\circ\text{C}$ , determine the average normal thermal stress developed in the bar.



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#### 4. Axial Load

#### EXAMPLE 4.10 (SOLN)

##### Equilibrium

As shown in free-body diagram,

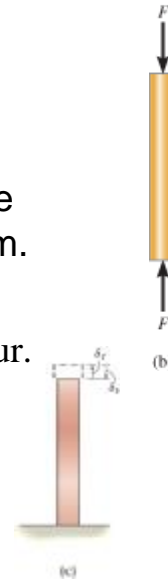
$$+\uparrow \sum Fy = 0; \quad F_A = F_B = F$$

Problem is statically indeterminate since the force cannot be determined from equilibrium.

##### Compatibility

Since  $\delta_{A/B} = 0$ , thermal displacement  $\delta_T$  at A occur. Thus compatibility condition at A becomes

$$+\uparrow \quad \delta_{A/B} = 0 = \delta_T - \delta_F$$



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#### 4. Axial Load

#### EXAMPLE 4.10 (SOLN)

##### Compatibility

Apply thermal and load-displacement relationship,

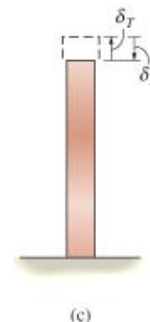
$$0 = \alpha \Delta TL - \frac{FL}{AL}$$

$$F = \alpha \Delta TAE = \dots = 7.2 \text{ kN}$$

From magnitude of  $\mathbf{F}$ , it's clear that changes in temperature causes large reaction forces in statically indeterminate members.

Average normal compressive stress is

$$\sigma = \frac{F}{A} = \dots = 72 \text{ MPa}$$



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#### 4. Axial Load

### CHAPTER REVIEW

- ❑ When load applied on a body, a stress distribution is created within the body that becomes more uniformly distributed at regions farther from point of application. This is the *Saint-Venant's principle*.
- ❑ Relative displacement at end of axially loaded member relative to other end is determined from

$$\delta = \int_0^L \frac{P(x) dx}{A(x) E}$$

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#### 4. Axial Load

### CHAPTER REVIEW

- ❑ If series of constant external forces are applied and  $AE$  is constant, then

$$\delta = \sum \frac{PL}{AE}$$

- ❑ Make sure to use sign convention for internal load  $P$  and that material does not yield, but remains linear elastic
- ❑ Superposition of load & displacement is possible provided material remains linear elastic and no changes in geometry occur

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#### 4. Axial Load

### CHAPTER REVIEW

- q Reactions on statically indeterminate bar determined using equilibrium and compatibility conditions that specify displacement at the supports. Use the load-displacement relationship,  $d = PL/AE$

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#### 4. Axial Load

### CHAPTER REVIEW

- q Change in temperature can cause member made from homogenous isotropic material to change its length by  $d = a\Delta TL$ . If member is confined, expansion will produce thermal stress in the member
- q If loading in bar causes material to yield, then stress distribution that's produced can be determined from the strain distribution and stress-strain diagram

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