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Equations for Mix Design of Structural Lightweight Concrete

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Abstract

Equations for mix design of structural lightweight concrete are presented. Conventionally, mix design of concrete is conducted using the tabular data and charts in standards. This requires extra efforts of understanding the data in the code and interpolations are often required when intermediate values are needed. The process is also liable to human error as data may be erroneously taken by the mix designer. The tabular data and graphs in ACI 211.2-98 are converted to equations. Various models were tried and the best model that adequately represents the data was chosen based on the regression coefficient and its predictive capability. The equations were used to solve some mix design problems from reputable textural sources. The developed equations are capable of giving material constituents for the first trial batch of structural lightweight concrete. These equations can be used in place of the data in the code and would reduce the effort, time and energy expended in the manual process of mix design of structural lightweight concrete. The equations are also useful for mixture proportioning adjustment.

Keywords: Concrete, Equations, Regression, Mix design

1. Introduction

Concrete mix design is the process of choosing suitable ingredients of concrete and determining their relative quantities with the objective of producing the most economical concrete while retaining the specified minimum properties such as strength, durability, and consistency (Akhras and Foo, 1994; Neville, 1995). The selection of ingredient is normally done using data from tables and charts in the relevant mix design standard. While these data and numerical examples in the codes are sufficient to guide the mix designer, it is thought worthwhile to add more values to these data for convenience of the users.

Researchers have supported the need to present mix design data in form of graphs or equations (Hover, 1995; Popovics, 1993). Most people involved in mix design of concrete may be more comfortable using equations to calculate the ingredient of concrete. The calculation of batch compositions using the mix design codes only give the first starting point (Neville, 1995; ACI 211.2-98; ACI 211.1-91; ACI 213-87). This is because the codes were developed from experience with certain materials in some parts of the world which may not be applicable for some materials in other parts of the world. The determination of accurate mix ingredient of concrete becomes more difficult when lightweight aggregates are used because of the problem associated with it such as high water absorption, lightweight, porous nature and surface texture. For this type of concrete it is obvious that searching for the optimum mix ingredients is quite a laborious task. Optimum compositions may be attained by testing of concrete, re-calculations and mix adjustment as deem necessary. This process can be made less cumbersome if the relevant equations are used for mixture proportioning.

The interest of this research is to use equations for mix design of structural lightweight concrete rather than the tabular data and graphs in the code to obtain the mix ingredient of structural lightweight concrete. The concrete mix designers do not have to scan through the numerous data in the code but instead deal with the variables in the equations. This may reduce the effort and time required to develop optimum mix compositions since adjustment can be done by choosing better input variables to improve on the mix composition.

2. Methodology

The tabular data and charts in ACI 211.2-98 (Standard practice for selecting proportions for structural lightweight concrete) are converted to equations. The equations were tested by determining the ingredients of structural lightweight concrete using sample problems from textural sources.

2.1. Mix Design Equations

Microsoft excel spread sheet was used to develop equations using the data in tables of ACI 211.2-98. The chart wizard in excel spread sheet was used to plot the graph. By right clicking on the data point and selecting ‘add trendline’ it is possible to choose any regression line to represent the data. The option tab in the ‘add trendline’ was used to display the regression equation and R-square value. Various regression models where tried and the trends of the graphs and correlation coefficients were used as the basis for selecting the best model that adequately represents the data. It was not possible to develop equations for choice of slump but the table is simple enough to choose the value of slump. The developed equations are shown as follows together with the numerical data that was used.

2.1.1. Mixing Water and Air Content
(i). Air-entrained Concrete

Table 1: Mixing water (kg/m^3) (Adapted from ACI 211.2-98)

Nominal Maximum Aggregate Sizes (mm)	Slump range (mm)		
	25-50	75-100	125-150
9.5	181	202	211
12.5	175	193	199
19	166	181	187

Mixing water
(125 to 150mm) slump,

$$y_1 = 0.2267 * x_1^2 - 8.9879 * x_1 + 275.92 \qquad (1)\ R^2 = 1$$

(75 to 100mm) slump

$$y_1 = 0.1215 * x_1^2 - 5.6721 * x_1 + 244.92 \quad (2)\ R^2 = 1$$

(25 to 50mm) slump

$$y_1 = 0.0648 * x_1^2 - 3.4251 * x_1 + 207.69 \quad (3)\ R^2 = 1$$

Table 2: Entrained air (%) (Adapted from ACI 211.2-98)

Nominal Maximum Aggregate Sizes (mm))	Condition of exposure		
	Mild exposure	**Moderate exposure**	**Extreme exposure**
9.5	4.5	6	7.5
12.5	4	5.5	7
19	4	5	6

Entrained air

(125 to 150mm) slump,

$$y_2 = 0.0013 * x_1^2 - 0.1964 * x_1 + 9.2436 \quad (4)\ R^2 = 1$$

(75 to 100mm) slump

$$y_2 = 0.0094 * x_1^2 - 0.3745 * x_1 + 8.7051 \quad (5)\ R^2 = 1$$

(25 to 50mm) slump

$$y_2 = 0.0175 * x_1^2 - 0.5526 * x_1 + 8.1667 \quad (6)\ R^2 = 1$$

(ii) Non-air-entrained concrete

Table 3: Mixing water (kg/m^3) (Adapted from ACI 211.2-98)

Nominal Maximum Aggregate Sizes (mm)	Slump range (mm)		
	25-50	75-100	125-150
9.5	208	228	237
12.5	199	217	222
19	187	202	208

Mixing water

(125 to 150mm) slump,

$$y_1 = 0.2996 * x_1^2 - 11.591 * x_1 + 320.08 \quad (7)\ R^2 = 1$$

(75 to 100mm) slump

$$y_1 = 0.143 * x_1^2 - 6.8138 * x_1 + 279.82 \quad (8)\ R^2 = 1$$

(25 to 50mm) slump

$$y_1 = 0.1215 * x_1^2 - 5.6721 * x_1 + 250.92 \quad (9)\ R^2 = 1$$

Table 4: Entrapped air (%) (Adapted from ACI 211.2-98)

NMAS (mm)	Entrapped air (%)
9.5	3
12.5	2.5
19	2

Entrapped air

$$y_3 = 0.0094 * x_1^2 - 0.3745 * x_1 + 5.7051 \quad (10)\ R^2 = 1$$

2.1.2. Water-cement ratio

Table 5: Water-cement ratio (Adapted from ACI 211.2-98)

Compressive strength (N/mm^2)	Non-air-entrained concrete	Air-entrained concrete
42	0.41	-
35	0.48	0.4
28	0.57	0.48
21	0.68	0.59
14	0.82	0.74

Non-air-entrained concrete

$$y_4 = -0.3749\ln(x_2) + 1.8148 \quad (11)\ R^2 = 0.999$$

Air-entrained concrete

$$y_4 = -0.3723\ln(x_2) + 1.7225 \quad (12)\ R^2 = 0.9999$$

2.1.3 Volume of oven-dry loose coarse aggregate per unit volume of concrete

Table 6: Volume of coarse aggregate per unit volume of concrete (Adapted from ACI 211.2-98)

Maximum size of aggregate (mm)	Fineness modulus			
	2.4	2.6	2.8	3
9.5	0.58	0.56	0.54	0.52
12.5	0.67	0.65	0.63	0.61
19	0.74	0.72	0.7	0.68

F.M = 2.4

$$y_5 = -0.002 * x_1^2 + 0.0745 * x_1 + 0.0546 \quad (13)\ R^2 = 1$$

F.M = 2.6

$$y_5 = -0.002 * x_1^2 + 0.0745 * x_1 + 0.0346 \quad (14)\ R^2 = 1$$

F.M = 2.8

$$y_5 = -0.002 * x_1^2 + 0.0745 * x_1 + 0.0146 \quad (15)\ R^2 = 1$$

F.M = 3.0

$$y_5 = -0.002 * x_1^2 + 0.0745 * x_1 - 0.0054 \quad (16)\ R^2 = 1$$

or in terms of the fineness modulus

Nominal maximum coarse aggregate size= 19 mm

$$y_5 = -0.1 * x_3 + 0.98 \quad (17)\ R^2 = 1$$

Nominal maximum coarse aggregate size= 12.5 mm

$$y_5 = -0.1 * x_3 + 0.91 \quad (18)\ R^2 = 1$$

Nominal maximum coarse aggregate size= 9.5 mm

$$y_5 = -0.1 * x_3 + 0.82 \quad (19)\ R^2 = 1$$

2.1.4. Weight of fresh lightweight concrete

Table 7: Weight of fresh lightweight concrete (kg/m3)(Adapted from ACI 211.2-98)

Specific gravity factor	Entrained air (%)		
	4	**6**	**8**
1	1596	1560	1519
1.2	1679	1643	1608
1.4	1768	1727	1691
1.6	1851	1810	1774
1.8	1934	1899	1857
2	2023	1982	1940

Entrained air = 4%

$y_6 = 426.14 * x_4 + 1169.3$ (20) $R^2 = 0.9999$

Entrained air = 6%

$y_6 = 423 * x_4 + 1135.7$ (21) $R^2 = 0.9999$

Entrained air = 8%

$y_6 = 419.29 * x_4 + 1102.6$ (22) $R^2 = 0.9999$

Specific gravity factor = 2.0

$y_6 = -20.75 * y_2 + 2106.2$ (23) $R^2 = 1$

Specific gravity factor = 1.8

$y_6 = -19.25 * y_2 + 2012.2$ (24) $R^2 = 0.9973$

Specific gravity factor = 1.6

$y_6 = -19.25 * y_2 + 1927.2$ (25) $R^2 = 0.9986$

Specific gravity factor = 1.4

$y_6 = -19.25 * y_2 + 1844.2$ (26) $R^2 = 0.9986$

Specific gravity factor = 1.2

$y_6 = -17.75 * y_2 + 1749.8$ (27) $R^2 = 0.9999$

Specific gravity factor = 1.0

$y_6 = -19.25 * y_2 + 1673.8$ (28) $R^2 = 0.9986$

2.1.5. Determination of Cement Content by Volume Method

The equations for cement content as a function of compressive strength were developed using figure 3.3.2 in ACI 211.2-98. The boundary region in this figure is a straight line, so linear equations were used to describe the feasible region of cement content.

For all-lightweight concrete, the following coordinate points along the boundary of figure 3.3.2 in ACI 211.2-98 were chosen:

Lower bound: A (27.60, 295.99), B (41.41, 413.98)

Upper bound: C (27.60, 393.46), D (41.41, 516.58)

For sand-lightweight concrete, the coordinate points chosen are:

Lower bound: A (20.7, 188.26), B (41.41, 367.81)

Upper bound: C (20.7, 308.815), D (41.41, 485.8)

The notation of the coordinate points takes the form (x_2, x_5).

Let the linear model be represented by

$$x_2 = a_0 + a_1 x_5 \quad (29)$$

Inserting the data for all-lightweight concrete in (29) yield

$$41.41 = a_0 + 413.98 a_1 \quad (30)$$

$$27.6 = a_0 + 295.99 a_1 \quad (31)$$

$$41.41 = a_0 + 516.58 a_1 \quad (32)$$

$$27.6 = a_0 + 393.46 a_1 \quad (33)$$

Solving equations (30) and (31) simultaneously gives

$a_0 = -7.04385$ and $a_1 = 0.117044$

and the minimum cement content is

$$x_{5L} = \frac{(x_2 + 7.04385)}{0.117044} \quad (34)$$

Similarly, equations (32) and (33) give $a_0 = -16.5291$ and $a_1 = 0.112159$ and the maximum cement content is

$$x_{5U} = \frac{(x_2 + 16.5291)}{0.112159} \quad (35)$$

Similar equations were developed for sand-lightweight concrete. The minimum and maximum cement contents are:

$$x_{5l} = \frac{(x_2 + 1.014677)}{0.115344} \quad (36)$$

$$x_{5U} = \frac{(x_2 + 15.408682)}{0.116959} \quad (37)$$

Where

y_1 = Water requirement (kg/m^3)

y_2 = Entrained air (%)

y_3 = Entrapped air (%)

y_4 = Water-cement ratio

y_5 = Volume of oven-dry loose coarse aggregate per unit volume of concrete (m^3)

y_6 = Weight of fresh lightweight concrete (kg/m^3)

x_1 = Nominal maximum coarse aggregate sizes (mm)

x_2 = Compressive strength (N/mm^2)

x_3 = Fineness modulus

x_4 = Specific gravity factor.

x_5 = cement content (kg/m^3)

a_0 = Intercept (regression coefficients)

a_1 = Slope (regression coefficients)

x_{5L} = Minimum cement content (kg/m^3)

x_{5U} = Maximum cement content (kg/m^3)

2.2. Choice of equations for mix design of concrete

The appropriate equations required for specific task in the mix design process are shown as follows.

2.2.1. Weight Method

This is applicable to mix design of sand-lightweight concrete. The procedure for the mix design is as follows

Step 1: Choice of Slump

Table 3.2.2.1 in the ACI code was used. It was not possible to provide equations for this table but the table is simple enough to enable users to decide on the required slump.

Step 2: Choice of Nominal Maximum Size of Coarse Aggregate

Recommended nominal maximum size of coarse aggregate by ACI 211.2-98 are: 9.5 mm, 12.5 mm and 19 mm.

Step 3: Estimation of Mixing Water and Air Content

Equations 1, 2, 3, 7, 8, 9 give the quantity of mixing water, while equations 4, 5, 6 and 10 give the quantity of air content.

Step 4: Selection of Appropriate Water-Cement Ratio

Equations 11 or 12 can be used.

Step 5: Calculation of Cement Content

This is the ratio of water content to water-cement ratio.

Step 6: Estimation of Lightweight Coarse Aggregate Content

Equations 13 to 19 can be used.

Step 7: Estimation of Fine Aggregate Content

Equations 20 to 28 can be used to obtain the weight of fresh lightweight concrete. Then the fine aggregate content is the total weight of fresh lightweight concrete less the weights of water, cement and coarse aggregate.

2.2.2. Volume Method

The volume method is used for mix design of sand-lightweight and all-lightweight concrete. Steps 1 (Choice of Slump), 2 (Choice of Nominal Maximum Size of Coarse Aggregate), and 3 (Estimation of mixing water and air content) are the same as in the weight method. The remaining steps of the mix design are as follows.

Step 4: Estimation of Cement Content

The minimum and maximum cement content can be obtained using equations (34) and (35) for all-lightweight concrete and equations (36) and (37) for sand-lightweight concrete.

Step 5: Estimation of Total Volume of Aggregate

The total volume of aggregates required (damp loose basis), as recommended by ACI 211.2-98, is between 1.04 m^3 to 1.26 m^3. It is at the discretion of the designer of the concrete mix to determine the required total volume of aggregate considering the nature of the lightweight aggregate and the properties of the concrete desired or recommendation from the manufacturer of the lightweight aggregate.

Step 6: Estimation of Loose Weight of Fine Aggregate

ACI 211.2-98 recommends the loose volume of fine aggregate to be between 40 to 60 percent of the total loose volume. The product of the loose volume of fine aggregate and the loose unit weight of fine aggregate gives the weight of fine aggregate.

Step 7: Estimation of Loose Weight of Coarse Aggregate

The total volume of concrete less the loose volume of fine aggregate gives the loose volume of coarse aggregate. The product of the loose volume of coarse aggregate and the loose unit weight of coarse aggregate gives the weight of coarse aggregate.

3. Application of the Derived Equations

The equations derived in this work were used to obtain the mix ingredient of concrete using sample mix design problems from reputable textural sources. The examples considered are as follows:

3.1. Case 1: Weight method

Design a concrete mixture by the weight method using lightweight coarse aggregate and normal-weight fine aggregate (sand-lightweight concrete) for structural lightweight concrete slab with a design 28-day compressive strength, f_c =35 Mpa. Use the following data in the mix design:

Coarse aggregate: 19mm – N0.4 size; specific gravity factor = 1.5; absorption = 11.0 %.
Fine aggregate: absorption = 1.0 %; fineness modulus = 2.80.
Oven dry loose weight of coarse aggregate = 769 kg/m^3 (Nawy, 2001).

3.2. Case 2: Volume method

A lightweight aggregate concrete containing normal weight fine aggregate is required to have a compressive strength (measured on standard cylinders) of 30 Mpa and a maximum air-dry density of 1700 kg/m^3. Compliance with the density requirement is determined using ASTM C 567-91. The required slump is 100 mm. The damp, loose density of the coarse and the fine lightweight aggregates is 750 and 880 kg/m^3, respectively. The normal weight fine aggregate has a density in a saturated and surface-dry condition of 1630 kg/m^3. From past experience, the required cement content for the trial mix can be taken as 350 kg/m^3. The volumes of aggregate to be used, in cubic metres per cubic metre of concrete, also chosen on the basis of experience, are: 0.60, 0.19, and 0.34, respectively, for the lightweight coarse, lightweight fine, and normal weight fine aggregate (Neville, 1995).

3.3. Numerical solutions

Case 1

1. Calculation of mixing water

From equation 2 x_1 = 19 mm.

$$y_1 = 0.1215 * x_1^2 - 5.6721 * x_1 + 244.92$$

$$y_1 = 0.1215 * (19)^2 - 5.6721 * (19) + 244.92 = 181 mm$$

2. Calculation of water-cement ratio

From equation 12 x_1 =35 Mpa

$$y_4 = -0.3723 \ln(x_2) + 1.7225$$

$$y_4 = -0.3723 \ln(35) + 1.7225 = 0.4$$

3. Cement content

$$Cement\ content = \frac{y_1}{y_4} = \frac{181}{0.4} = 452.5 kg/m^3$$

4. SSD weight of coarse aggregate

The volume of oven dry loose coarse aggregate is given by equation (15)

$$y_5 = -0.002 * x_1^2 + 0.0745 * x_1 + 0.0146$$

$$y_5 = -0.002 * (19)^2 + 0.0745 * (19) + 0.0146 = 0.7 m^3$$

$$Dry\ weight\ of\ coarse\ aggregate = 0.7 * 769 = 538.3 kg/m^3$$

$$SSD\ weight\ of\ coarse\ aggregate = 1.11 * (538.3) = 597.51 kg/m^3$$

5. SSD weight of fine aggregate

The weight of fresh concrete is obtained using equation 21 with x_4 = 1.5.

$$y_6 = 423*x_4 + 1135.7$$

$$y_6 = 423*(1.5) + 1135.7 = 1770.2\,kg/m^3$$

$$SSD\ weight\ of\ \ fine\ aggregate = 1770.2 - (181 - 452.5 - 597.51) = 539.19\,kg/m^3$$

Case 2

The solution to this problem is quite easy, but the derived equations will furnish options for possible cement content and water content if these are not given.

For x_2 = 30 N/mm^2, the cement content is calculated as follows:

From equation 36, the minimum cement content is

$$x_{5l} = \frac{(x_2 + 1.014677)}{0.115344}$$

$$x_{5l} = \frac{(30 + 1.014677)}{0.115344} = 268.89\,kg/m^3$$

From equation 37, the maximum cement content is

$$x_{5U} = \frac{(x_2 + 15.408682)}{0.116959}$$

$$x_{5U} = \frac{(30 + 15.408682)}{0.116959} = 388.24\,kg/m^3$$

The cement content assumed by the author, 350 kg/m^3, is within the range found here.

The mixing water required (for air-entrained concrete) is calculated using equation 2, x_1 = 19 mm.

$$y_1 = 0.1215*x_1^2 - 5.6721*x_1 + 244.92$$

$$y_1 = 0.1215*(19)^2 - 5.6721*(19) + 244.92 = 181\,kg/m^3$$

Lightweight coarse aggregate $= 0.63*750 = 472.5\,kg/m^3$

Lightweight fine aggregate $= 0.19*880 = 167.2\,kg/m^3$

Normal weight fine aggregate $= 0.34*1630 = 554.2\,kg/m^3$

The results of mix design using equations were compared with those found in text books as indicated in table 8. The mix compositions calculated using the equations agree reasonably well with those of the two authors. The slight disparity is due to approximations in the course of the calculations.

Table 8: Results of first trial batch

Mix Ingredient	Equation	Case 1
Weight method		
Mix Ingredient	**Equation**	**Case 1**
Water (kg/m^3)	181	180.96
Cement (kg/m^3)	452.5	452.69
Fine aggregate , SSD (kg/m^3)	539.19	538.12
Coarse aggregate, SSD (kg/m^3)	597.51	597.45
Water/cement ratio (%)	0.4	0.4
Volume method		
Mix Ingredient	**Equation**	**Case 2**
Water (kg/m^3)	181	180.00
Cement (kg/m^3)	350	350
Lightweight fine aggregate (kg/m^3)	167.2	168
Normal-weight fine aggregate (kg/m^3)	554.2	550
Coarse aggregate, SSD (kg/m^3)	472.5	473
Water/cement ratio (%)	0.52	0.51

4. Summary

The equations presented in this work are capable of giving the material constituents of structural lightweight concrete for the first trial batch from given performance criteria. Mix adjustment of the ingredients can be made simply by choosing suitable input variables and re-calculating the batch composition. The equations can be updated when new version of the code is developed by developing new equations to replace the once presented in this work. Interpolations are avoided if equations are used. For example, in the example of the weight method, the weight of fresh concrete can be obtained by interpolating between the tabulated weights for specific gravity factors of 1.4 and 1.6. Using an equation, the weight of fresh concrete was calculated directly. Addition of equations to future versions of mix design codes may add more value to the existing documents.

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